The Perceived Sources of Unexpected Inflation *

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Abstract

We use high-frequency asset price changes around Consumer Price Index announcements in the US to learn about market perceptions regarding the economy. We write a New Keynesian Model with incomplete information and an inflation announcement to extract the demand and supply share of unexpected inflation through observable asset price changes around the announcement. The key intuition is that, given a standard Taylor rule, if consumption expectations rise in response to a positive surprise in inflation, it implies that a positive demand shock plays an important role, whereas if consumption expectations fall in response, it highlights the significance of a negative supply shock. Empirically, we find that the response of expected future annual dividends of S&P 500 companies to a positive surprise in inflation around US CPI announcements was positive before the Covid period but turned negative post-Covid. We use these to construct high-frequency changes in expectations of future real consumption. We also find that future treasury nominal yields and forward breakeven inflation rates increase in response to a positive surprise in inflation throughout the period. Interpreting our empirical findings through the lens of the model, we find that the relative share of supply in unexpected inflation has increased by 20 percentage points post-Covid.

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Keywords: inflation, high-frequency, demand, supply, asset prices, expectations

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1 Introduction

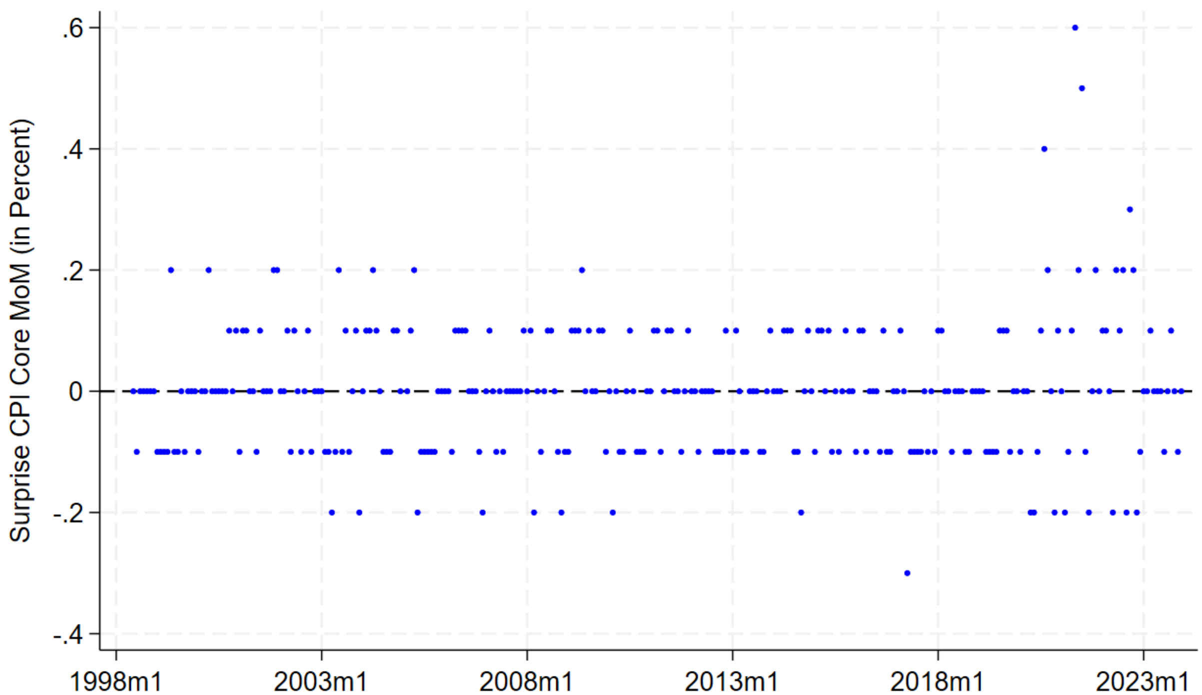
Large asset price movements occur around macroeconomic news releases, and this is well-documented in the literature. However, how these asset price movements reflect what the market actually learns regarding the macroeconomy, for example, the demand or supply side, has been relatively less explored. One application is to study asset price movements around Consumer Price Index (CPI) announcements in the US to understand the sources of inflation as perceived by the market.

There is a recent strand of literature that tries to understand the sources of inflation during and after the pandemic. For example, some papers highlight the role of fiscal spending in creating demand-driven inflation (for instance, Giannone and Primiceri (2024)), while others emphasize the role of supply chain bottlenecks due to the pandemic and the war in creating supply-driven inflation (for example, Bai et al. (2024)). Central bankers frequently reference "demand" and "supply" conditions in FOMC speeches, claiming that the current and future policy path response differs based on the type of the shocks [1]. Our paper aims to elucidate how the market decomposes its forecast errors of inflation to demand and supply. This is useful because, first, market perception of sources of inflation is useful for policymakers to learn from and to anticipate market's response to different policies. Second, and more important, there have been massive forecast errors or surprises in inflation in the last few years (as high as three times the average value of inflation), thus making it useful to study the sources of this unexpected component of inflation (see Figure 1).

The key intuition for distinguishing between demand and supply shocks is as follows. According to textbook economic theory, an increase in both prices and quantities typically indicates a strong demand shift. Conversely, if prices rise while quantities fall, it suggests a relatively strong supply shift. Similarly, if a positive surprise in inflation (i.e., the price) leads to an increase in expected real consumption (i.e., the quantity), it suggests that a positive demand shock plays a significant role in unexpected inflation. On the other hand, if consumption expectations decrease in response, it points to a significant role of negative supply shocks. Thus, this co-movement between expected real consumption and surprise in inflation can help identify the dominant shock. However, to quantify the degree to which each shock is driving inflation, we need a model. The model will also shed light on the mechanism behind how a macroeconomic news release about the past influences forward-looking asset prices.

[1] https://www.federalreserve.gov/newsevents/speech/powell20231109a.htm

Figure 1: **Time Series of CPI Surprises**



This figure plots the time series of $surprise_t^{CPI}$ which is the difference between the CPI Core Month on Month actual or announced value in each month and the CPI Bloomberg Survey Median (in percent) from 1998m6 to 2023m12.

We write a New Keynesian model with Calvo pricing and Taylor rule to interpret asset price movements around CPI announcements. Both firms and the representative household have incomplete information. The key new element added to this model is an aggregate inflation announcement. The model mechanism is as follows. There are two shocks in the model: a demand shock, which is represented as a discount factor shock, and a supply shock, which can be interpreted as a markup shock, a labor supply shock, or a technology shock. First, firms set their prices based on their dispersed information about both shocks. Thus, the aggregate measure of inflation is a combination of the underlying demand and supply shocks. When this measure is announced, it acts as an additional signal of the underlying two shocks. The market participants, i.e., the representative household, have incomplete information about the two shocks and, therefore, use the inflation signal to revise their beliefs about the underlying shocks by Bayes' rule.

According to Bayes' rule, there are two main factors that affect the revision of beliefs of both shocks. There is a larger revision of belief about a shock if the shock has a larger impact on inflation. Additionally, these revisions are larger when the household has more uncertain prior beliefs. For instance, if there is an unprecedented fiscal policy and the market is unsure about demand conditions, a positive surprise in inflation will lead to a stronger upward revision of the positive demand shock. Similarly, if a war in Ukraine begins and market uncertainty about supply chain issues increases, a positive surprise in inflation will result in a stronger revision of a negative supply shock.

Revising beliefs about current shocks also influences expectations about the future state of the economy, given the persistence of these shocks. This, in turn, alters expectations about future interest rates and consumption, ultimately impacting forward-looking asset prices. There are two unobservables in the economy: the revision of belief about the demand shock and the revision of belief about the supply shock. To extract these using the model, we focus on two asset price changes around the CPI announcement, the nominal treasury yields and the dividend futures of S&P 500 companies. The changes in expectations about future interest rates (due to the inflation announcement in the model) are measured using changes in treasury yields around the CPI announcement. On the other hand, the changes in expectations about future real consumption are mapped to a measure constructed using dividend futures.

We document how asset prices change around CPI announcements. We find that, on average, the nominal treasury yields of all horizons increase in response to a positive surprise in inflation. This is unsurprising, as the market expects the Fed to increase rates to fight inflation. Apart from that, expectations of future real dividends of S&P 500 companies, as implied by dividend futures and breakeven inflation rates, rose in response to a positive surprise in inflation from 2016 to 2019 but fell from 2020 to 2023. Since this is used to construct a measure of expectation regarding future real consumption, qualitatively, this highlights the increasing role of supply shocks post-Covid on an average.

We construct a formula to calculate the share of each shock in unexpected inflation. It depends on parameter values of the New Keynesian model, which are assumed to be constant and taken directly from the literature, and the revision of beliefs about both the shocks, which are assumed to be time-varying and inferred from the mapping exercise explained in the previous paragraph. Since these time-varying revisions of beliefs are inferred from asset price movements that are noisy, we take rolling averages in asset price changes to address this issue. We find that the relative share of supply in unexpected inflation increased by 20 percentage points from 2016-2019 to 2020-2023.

Related Literature. This paper broadly contributes to three strands of literature.

There is a rapidly growing literature that studies the sources of the recent inflation surge. One type of literature tries to understand actual sources of inflation. The paper closest to us is that of Shapiro (2022) which uses the underlying category level data of the personal consumption expenditures index (PCE) and similar price quantity co-movement identification to decompose inflation into demand and supply components. They find substantial role of both demand and supply shocks in post-pandemic inflation. Giannone and Primiceri (2024), on the other hand, find that the inflation was driven by unexpectedly strong demand forces. Some papers such as Bai et al. (2024) highlight the causal impact of supply chain disruptions on inflation. Some other relevant papers on the recent actual sources of inflation that shaped our perspective are Bernanke and Blanchard (2023), Gagliardone and Gertler (2023), Rubbo (2023), Comin et al. (2023), Schmitt-Grohé and Uribe (2022), Benigno and Eggertsson (2023) and Acharya et al. (2023). Our study contributes to this literature by shifting the focus from actual to perceived sources of inflation, providing a novel perspective on how market participants interpret inflationary pressures. Our findings indicate that, during and after Covid-19, market participants viewed supply chain pressures as important drivers of the inflationary forces.

Another type of literature tries to study market perceptions regarding sources of inflation. Some papers that use asset prices to focus on perceived sources of inflation are Cieslak and Pflueger (2023) and Pflueger (2023). They show that different types of shocks should generate different bond-stock correlations on an average over a period of time through the lens of a New Keynesian Model. We, on the other hand, specifically focus on asset price changes around CPI announcements to get the causal impact of this information release on market expectations.

There is a vast literature on the reaction of financial markets to macroeconomic news. McQueen and Roley (1993), Faust et al. (2007), Savor and Wilson (2013), Ai and Bansal (2018), Fisher et al. (2022), Gil de Rubio Cruz et al. (2023) and Bocola et al. (2024) are few of the papers that influenced our thinking. There is also a huge literature on the impact of monetary policy announcements on asset prices starting from Cook and Hahn (1989) and Kuttner (2001). Mertens and Zhang (2023) also does an event study analysis to understand how New Keynesian parameters change around announcement. We contribute to this literature by specifically modelling announcement in a New Keynesian model. We interpret the asset price reaction to announcement results through the model to extract the belief updates in the underlying macro-fundamental shocks.

Finally, some parallels can be drawn with the literature that tries to decompose a monetary policy announcement into monetary versus non-monetary news. Melosi (2017), Nakamura and Steinsson (2018), and Jarociński and Karadi (2020) were particularly relevant. Our paper, instead, decomposed macroeconomic news releases into demand versus supply shock components.

The rest of the paper is organized as follows. Section 2 writes a model. Section 3.1 describes the data. Section 3.2 provides summary statistics and empirical methodology and results. Section 4 combines the empirical results and the model to discuss findings. Section 5 concludes with the next steps.

2 New Keynesian Model with Incomplete Information and Announcement

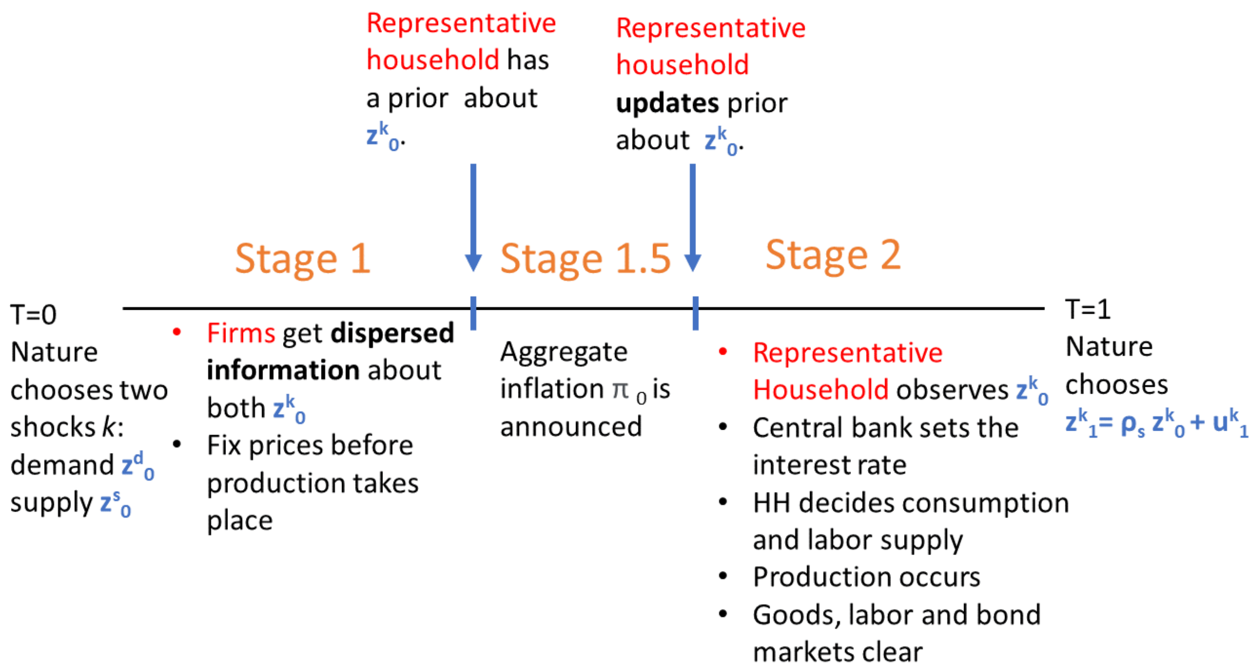
In this section, we write a New Keynesian model with incomplete information and an aggregate inflation announcement. The model is useful for understanding why a macroeconomic news release about the past could affect forward-looking asset prices and for decomposing the surprise in the release to the underlying shocks. Apart from that, the model also helps quantify the rise in the share of supply in inflation post-Covid, beyond the intuition of the co-movement of consumption expectations and the surprise in inflation.

We build on a basic New Keynesian model as in Woodford (2003) and Gali (2003) chapter 3. The first key new element in our model is an announcement of aggregate inflation. This is important for understanding how beliefs about the present and the future get updated when this announcement takes place, and how this can affect forward-looking asset prices. It is the asset price movements in the data around the announcement that will eventually be used to discipline the model and recover the changes in beliefs.

The second crucial assumption is incomplete information on the representative household. If the household has complete information about the underlying shocks, then it should be able to forecast exactly what the aggregate inflation is going to be. Thus, the announcement of inflation will create no surprise, and have no bite regarding beliefs about the present and the future, and thus cause no asset price movements around such an announcement. Since this is not what happens in reality, we require the household to have incomplete information. The third assumption that we incorporate is that the firms have dispersed information regarding the underlying structural shocks. This assumption is not crucial for the mechanism of the model and nests the standard case where the firms know the shocks perfectly. We model it in this way to capture the fact that firms might have more precise information regarding supply as compared to demand conditions. This also helps us match the model to the data better without losing tractability but does not change results qualitatively.

The model timeline is shown in Figure 2. At time $t = 0$, nature chooses two shocks in the economy: the demand shock and the supply shock. In the first stage, which we call stage 1, the firms producing differentiated goods have dispersed information about the underlying shocks and fix their prices based on that before any production takes place. Calvo rule applies, so only a fixed fraction of firms are allowed to change their prices. Now that prices are set, a measure of aggregate inflation exists, and in the next stage, which we call stage 1.5, this measure is announced. In the final stage, which we call stage 2, all agents can now observe what the shocks are. The central banks sets the interest rate by Taylor rule. The representative household can now observe all the prices and will optimally choose consumption, saving, and labor supply. Production occurs to satisfy demand, and wages adjust to clear all three markets: bond, labor, and goods market.

Figure 2: Timeline of New Keynesian Model with incomplete information and inflation announcement



Fundamentals and information. We have two shocks in our economy: discount factor shock that we call the demand shock z_t^d, and a marginal cost shock, which could be interpreted as a markup, technology or labor supply shock z_t^s. The law of motion of the demand and supply shock z_t^k is given by an AR(1) process:

$$z_t^k = \rho_k z_{t-1}^k + u_t^k \tag{1}$$

where $u_t^k \sim \mathcal{N}(0, \sigma_{k0}^2)$ and $\rho_k \in (0,1)$. The normality assumption is for convenience in solving the model. The AR(1) process is chosen (instead of an i.i.d process, for example) to capture the empirical fact that future interest rates and inflation measured from treasury yields and forward breakeven inflation rates also respond significantly to a surprise in inflation. This is elaborated later in the empirical facts section 3.2 and in Figures 5 and 6. We interpret these facts as evidence of the persistence of these shocks.

The aggregate fundamentals of the economy in period t are identified by the joint distribution of the shocks $u_t \equiv (u_t^d, u_t^s)$. Firms have dispersed information about shock u_t in stage 1. To elaborate, firms j

have dispersed signals about the underlying shocks $k \in (d, s)$

$$x_{jt}^k = z_t^k + u_{jt}^k$$

where $u_{jt}^k \sim \mathcal{N}(0, \sigma_k^2)$. If $\sigma_k^2 \to 0$, then we reach the nested situation where all firms have perfect information about u_t in stage 1. We add dispersed information on the side of the firms to allow for the fact that these price-setting firms could have different precision of information regarding demand or supply shocks. The household needs to observe u_t as well as the all the prices set by firms only in stage 2 to make its optimal consumption-saving decision. The Fed also sets the interest rate i_t in stage 2. By stage 2, all the agents know the shock realization $u_t \equiv (u_t^d, u_t^s)$ with complete certainty. The timeline with the shocks can be represented as in Figure 2.

Stage 2. The model will be solved by backward induction and hence we start with the last stage. Since the household (as well as all other agents) observes the shocks u_t perfectly in stage 2, as in a standard New Keynesian model, its total utility optimization exercise will yield the familiar log-linearized Euler equation (for microfoundations see A.1 or Gali (2003) Chapter 3).

$$c_t = \mathrm{E}_t[c_{t+1}] - \frac{1}{\gamma}(i_t - \mathrm{E}_t[\pi_{t+1}]) + z_t^d \tag{2}$$

z_t^d is the demand shock, and γ is the inverse of intertemporal elasticity of consumption. The interest rate is set by the Fed's Taylor rule

$$i_t = \phi_\pi^{Tay} \pi_t + \phi_y^{Tay}(y_t - y_t^n) \tag{3}$$

where $y_t^n \equiv -z_t^s/(\gamma + \psi)$ is the natural level of output. z_t^s is the supply shock, and ψ is the Frisch elasticity of labor supply. ϕ^{Tay} are the Taylor coefficients. We could add a monetary shock to the model, but as long as the firms and households have the same information about the monetary shock, we can show that it will not matter for our purposes. So, we are refraining from including it here. In section 4.4, we will talk more about how monetary policy shocks can be interpreted in our model. The Taylor rule captures well the empirical fact stated in the section 3.2 that nominal yields increase with a positive surprise in inflation.

Stage 1. Now, we discuss the dynamics in stage 1. We will discuss stage 1.5 later as we will show that it has no impact on real allocations. The firms anticipate the household and Fed behavior in stage 2, and accordingly fix prices in stage 1 to maximise profits. We assume the standard assumption of Calvo pricing in NK models, that is, only $1-\theta$ firms are allowed to change their prices every period.

If firm j is allowed to reset their price, they will choose the optimal price $p_t^*(j)$ that will maximise their present discounted value of current and future profits. Let $\pi_t^*(j) = p_t^*(j) - p_{t-1}$ be called the optimal reset inflation for firm j. In log-linearized form, the optimization exercise gives

$$\pi_t^*(j) = (1-\beta\theta)E_{jt}\hat{mc}_t + E_{jt}\pi_t + \beta\theta E_{jt}\pi_{t+1}^*(j) \tag{4}$$

where mc_t is the marginal cost and $\hat{mc}_t = (\gamma+\psi)y_t + z_t^s$, and β is the discount factor. So, a firm's optimal reset inflation depends on its expected current marginal cost, expected current aggregate inflation, and its expected future optimal reset inflation. Thus, a firm will optimally raise its prices if given a chance, if its current real marginal cost is higher, or if the current or future inflation is higher, as it is not allowed to change its prices in every period. Future real marginal costs are implicitly taken into account through future optimal reset inflation. This is exactly how a firm sets its price in a standard New Keynesian model as in Gali (2003), except now it is firm-specific expectations E_{jt} instead of E_t which is supposed to represent the same expectations for all firms.

Now, at the beginning of time $t+1$, firms will be identical because all the shocks are visible to all the firms at the end of the period t. Thus, $\pi_{t+1}^*(j) = p_{t+1}^*(j) - p_t$ is exante (at period t) expected to be the same for all j and is equal to reset inflation averaged across all the firms π_{t+1}^*. The average reset inflation is given by

$$\pi_t^* = \int_j \pi_t^*(j)\,dj \tag{5}$$

Since only $(1-\theta)$ fraction of randomly chosen firms can change their prices, the aggregate inflation will be $1-\theta$ times average reset price of all firms i.e.,

$$\pi_t = (1-\theta)\pi_t^* \tag{6}$$

Note that if all firms have perfect information about the shock u_t in stage 1, i.e, $\sigma_k^2 \to 0$ for all $k \in (d,s)$

then we get the familiar Phillips curve as in a standard NK model

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \frac{(1-\theta)(1-\beta\theta)}{\theta} \hat{mc}_t \tag{7}$$

Stage 1.5: Adding Announcement. At stage 1, the firms fix their prices. So an aggregate measure of inflation exists. However, at this stage, the households have incomplete information about the value of the shock $u_t \equiv (u_t^d, u_t^s)$, and cannot observe the prices set by the firms as well. They can observe both u_t and the prices only in stage 2. Between stage 1 and stage 2, we add another stage, called stage 1.5, where the only activity that takes place is an announcement of aggregate inflation (see Figure 2). The aggregate inflation is an additional source of information regarding the underlying fundamentals u_t, and the household accordingly updates its belief regarding them. Importantly, this has no impact on real allocations in our model since the real allocations take place in stage 2 when all shocks are fully visible. This is by design, since we want to understand how beliefs are updated, not how they impact real allocations. Thus, the Euler equation, modified Phillips curve and Taylor rule remain unchanged.

In real life, when firms set prices, the households can immediately observe them if they want to. However, all households do not purchase all goods, and thus they are unaware of aggregate inflation but only have incomplete information. Having multiple households with dispersed information about the prices would come at the cost of tractability. Thus, we make the simplifying assumption of a representative household which has incomplete information about the underlying structural shocks and cannot observe all the individual prices in stage 1.

Model Equilibrium Solution. As is usual in these kind of models, the equilibrium levels of aggregate output, inflation, and interest rates can be given by a linear combination of the structural shocks (ignoring constants)

$$c_t = \boldsymbol{a_c} \cdot \boldsymbol{z_t} \tag{8}$$

$$\pi_t = \boldsymbol{a_\pi} \cdot \boldsymbol{z_t} \tag{9}$$

$$i_t = \boldsymbol{a_i} \cdot \boldsymbol{z_t} \tag{10}$$

where $\boldsymbol{a_c} \equiv (a_c^d, a_c^s)'$, $\boldsymbol{a_\pi} \equiv (a_\pi^d, a_\pi^s)'$ and $\boldsymbol{a_i} \equiv (a_i^d, a_i^s)'$ can be calculated in terms of model parameters. $\boldsymbol{z_t}$ is the vector of shocks, $\boldsymbol{z_t} \equiv (z_t^d, z_t^s)'$.For the full model equilibrium solution, please look at Appendix A.1.

Furthermore, as expected, a positive demand shock i.e., $z_t^d > 0$ leads to an increase in consumption, inflation and nominal interest rate i.e., $a_c^d > 0, a_\pi^d > 0, a_i^d > 0$ and a negative supply shock i.e., $z_t^s > 0$ leads to an decrease in consumption and an increase in inflation and nominal interest rate i.e., $a_c^s < 0, a_\pi^s > 0, a_i^s > 0$. If firms had perfect information when setting prices, then they would perfectly anticipate how the households would behave in stage 2. In that case, these weights **a** would be the same as in a standard New Keynesian model with complete information. The dispersed information lessens the impact of the shocks on inflation and, subsequently, interest rates and quantities as firms are less certain about the value of the shock. However, this does not have any impact qualitatively. Quantitatively, since we assume firms have more precise information regarding supply conditions as compared to demand conditions in the model matching section 4, this puts some downward pressure on the share of demand in unexpected inflation both before and after Covid. For the exact formula, please look at Appendix A.1.

2.1 Announcement and Revision of Beliefs

In this subsection, we talk about the interesting stage 1.5, which is vital to understanding why and how beliefs change around macroeconomic news releases. At the end of stage 1 (before the announcement of aggregate inflation in stage 1.5), the household's prior about the underlying demand and supply shock z_t^k are assumed to be normally distributed with mean and variance given by

$$\mu_{ht}^k \equiv \mathbb{E}_{ht}^{ba}\left[z_t^k\right]$$

$$\sigma_h^{2k} \equiv \mathbb{V}_{ht}^{ba}\left[z_t^k\right]$$

where $\mathbb{E}_{ht}^{ba}$ and $\mathbb{V}_{ht}^{ba}$ refers to expectations and variance of household (h) prior beliefs before announcement (ba) at time period t respectively. The normality assumption is for simplicity. We are not taking a stance on what the value of μ_{ht}^k and σ_h^{2k} are. If the household receives no extra information or signals regarding the underlying shocks in stage 1, then their prior before announcement would simply be based on the actual underlying law of motion (1), i.e. $\mu_{ht}^k = \rho_k z_{t-1}^k$ and $\sigma_h^{2k} = \sigma_{k0}^2$. If the household receives additional signals in stage 1, then μ_{ht}^{k} and σ_{h}^{2k} would change accordingly. Also, no time subscript is given for the variance of household prior beliefs since it is assumed to be the same in every period for simplicity.

From (9), since inflation is a linear combination of the underlying shocks in equilibrium, the household's prior belief about mean and variance of aggregate inflation price at the end of stage 1 before announcement is given by

$$\mu_{ht}^{\pi} \equiv \mathbb{E}_{ht}^{ba}[\pi_t] = a_{\pi}^{d}\mu_{ht}^{d} + a_{\pi}^{s}\mu_{ht}^{s}$$

and the variance is given by

$$\sigma_{h}^{2\pi} \equiv \mathbb{V}_{ht}^{ba}[\pi_t] = a_{\pi}^{2d}\sigma_{h}^{2d} + a_{\pi}^{2s}\sigma_{h}^{2s}$$

Also, the covariance of household prior between the shock and inflation is given by

$$cov_h(\pi, z_t^k) = a_{\pi}^{k}\sigma_{h}^{2k}$$

Now we focus on what happens to household's beliefs after the inflation announcement when they follow Bayes' rule.

Proposition 1. *The household's change in belief about shock k after π_t is announced (as say $\bar{\pi}_t$) is given by*

$$\underbrace{\mathrm{E}_{ht}\left[z_t^k|\pi_t = \bar{\pi}_t\right] - \mathrm{E}_{ht}^{ba}\left[z_t^k\right]}_{\Delta^{ann}E_{ht}z_t^k} = \frac{cov_h(\pi, z_t^k)}{\sigma_h^{2\pi}}(\bar{\pi}_t - \mu_{ht}^{\pi}) = \frac{a_{\pi}^{k}\sigma_{h}^{2k}}{\sigma_{h}^{2\pi}}\underbrace{(\bar{\pi}_t - \mu_{ht}^{\pi})}_{surprise_{ht}^{\pi}} \tag{11}$$

Proof. We know that

$$\begin{pmatrix} z_t^k \\ \pi_t \end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix} \mu_{ht}^{k} \\ \mu_{ht}^{\pi} \end{pmatrix}, \begin{pmatrix} \sigma_h^{2k} & cov_h(\pi, z_t^k) \\ cov_h(z_t^k, \pi) & \sigma_h^{2\pi} \end{pmatrix}\right)$$

Then, by property of bivariate normal distribution, the conditional distribution of shock z_t^k given π_t is announced as $\bar{\pi}_t$ is given by

$$z_t^k \mid (\pi_t = \bar{\pi}_t) \sim \mathcal{N}\left(\mu_{ht}^k + \frac{cov_h(\pi, z_t^k)}{\sigma_h^{2k}}\left(\bar{\pi}_t - \mu_{ht}^{\pi}\right), \left(1 - \left(\frac{cov_h(\pi, z_t^k)}{\sigma_h^{\pi}\sigma_h^k}\right)^2\right)\sigma_h^{2k}\right).$$

Thus,

$$\mathrm{E}_{ht}(z_t^k \mid (\pi_t = \bar{\pi}_t)) - \mathrm{E}_{ht}[z_t^k] = \frac{cov_h(\pi, z_t^k)}{\sigma_h^{2k}}\left(\bar{\pi}_t - \mu_{ht}^{\pi}\right)$$

where $\mathrm{E}_{ht}\left[z_t^k\right] = \mu_{ht}^k$. Hence proved. □

Equation 11 is quite intuitive. It is reminiscent of an OLS regression of z_t^k on π_t where the OLS coefficient is $\frac{cov_h(\pi, z_t^k)}{\sigma_h^{2\pi}}$. Intuitively, the agents make the *best linear prediction* of z_t^k given $\pi_t = \bar{\pi}_t$. Otherwise as well, the equation makes intuitive sense. The higher the surprise in the aggregate inflation, the higher the revision in belief of underlying fundamental shocks, i.e., all updates in expectations about the structural shocks are directly proportional to the surprise in inflation. Second, the more the weight of a particular shock in the equilibrium inflation, i.e., higher the value of a_π^k, higher is the revision of belief about shock k. This essentially means that if a particular shock does not impact inflation much, then the inflation announcement will not lead to much revision of that shock. Finally, if households receive less precise signals about the shock k, i.e., a high variance σ_h^{2k}, then they update more their beliefs about that shock. For example, Covid and the Ukraine war created more uncertainty regarding supply chains, leading to poorer information about supply related shocks. In that case, if a higher than expected inflation is realized, then the household is going to revise its belief more about the supply shock since it was more uncertain about it. Similarly, an unprecedented fiscal policy will create more uncertainty about demand conditions, leading to more revision of demand shocks in case of a surprise in inflation. Thus, the revision of the underlying shocks will depend on how uncertain we are about the underlying shocks.

The idea is that firms have some information about underlying shocks in stage 1 while setting prices. Thus, the announced inflation is an additional signal of the underlying shocks for the households, and they try to infer the value of the underlying shocks using this extra information. Note that in this dispersed information setting, the quality of information that the individual firms receive for each of the shocks is taken into account by the household in its belief update process. For example, if the information that firms receive for demand is more dispersed and thus less informative, it will weaken the coefficient a_π^d and thus the covariance between inflation and the demand shock. Thus, in that case, the household will infer less about demand shocks from the inflation announcement.

The next section describes the data that can be used to discipline the model, specifically the announcement stage.

3 Empirical Methodology and Results

In this section, we will discuss the data sources used and the high-frequency empirical methodology and results.

3.1 Data

In this section, we describe the data sources. We use the expectations of CPI from Bloomberg and high-frequency changes in asset prices from the Federal Reserve, Chicago Mercantile Exchange, Tickdata and Bloomberg.

Bloomberg survey: Bloomberg surveys academics, professionals from banks, finance, etc. regarding their forecast, or more appropriately "backcast", for the past month's CPI. The data is available in their "ECO" function. According to Bloomberg, "surveys listed on the ECO function normally start one to two weeks before a release, and are updated on a constant, real-time basis leading up to that release". Thus, if the CPI for October 2023 is announced on November 14, 2023 at 8:30 AM EST, then the survey opens up around November 1, 2023, and the surveyees are allowed to update their "backcasts" until 8:29 AM EST on November 14, 2023. Thus, any information released until the announcement could be taken into account by the surveyees in forming their "backcasts". The variables included in this dataset are the date and time of announcement, the event that is announced and the announced value (the original source being **Bureau of Labor Services** for that), the survey median, high, low, average and standard deviation of the "backcast" and number of people surveyed. We specifically focus on the data from 2004-2023 and the data is at monthly frequency. CPI monthly announcements became scheduled regularly 2004 onwards and 2023 is the end of our sample.

Asset price data: We use the tick-level Treasury futures data (tickers TU, FV and TY) from the Chicago Mercantile Exchange and tickdata.com. We calculate the implied yields using the corresponding duration from Bloomberg [2]. We use data about the daily nominal treasury yields [3] and real treasury yields[4] from the Fed. We collect data on daily dividend futures and stock price of S&P 500 from Bloomberg.

Consumption: We collect monthly nominal aggregate consumption expenditures on non-durables and services from National Income and Product Accounts (NIPA) Table 2.8.5.

3.2 Summary Statistics and Empirical Results

Our goal in this section is to show summary statistics about inflation surprises and document facts about the effect of these inflation surprises of scheduled consumer price index (CPI) announcements on the bond and stock market. These empirical results will eventually be used to discipline the model written in section 2.

We look at the seasonally adjusted CPI for all urban consumers. We define a surprise in CPI inflation as follows:

$$surprise_t^{CPI} \equiv CPI_t^{ann} - \mathrm{E}[CPI]_t^{Bloomberg} \tag{12}$$

where CPI_t^{ann} is the CPI announced in period t for the past period or month $t-1$ (in percentage form) and $\mathrm{E}[CPI]_t^{Bloomberg}$ is the median expectations of the surveyees of Bloomberg in period t regarding the CPI of the past month $t-1$. The CPI announcement and the survey are for both core goods (excluding food and energy, referred to as CPI Core) and all the goods (referred to as simply CPI) in the basket. We focus on CPI month-on-month (MoM) announcement and surprise because conceptually, CPI month-on-month surprise is easier to interpret within a model where each time period is a month, and inflation news about the month is revealed in the announcement. However, the results are similar for CPI year-on-year (YoY) as well. While studying asset price movements around these announcements, we will especially focus on CPI Core MoM surprises to avoid the volatility associated with food and energy prices, although the results are similar for non-core surprises.

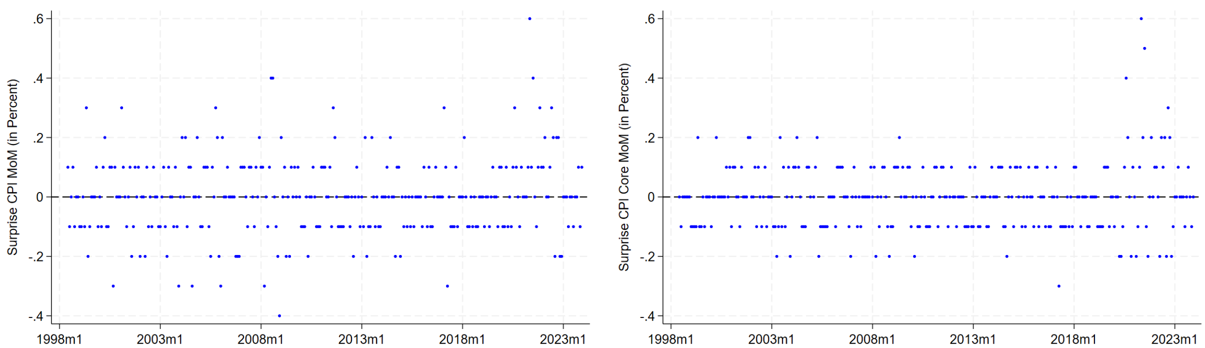
We plot the time series of CPI MoM and CPI Core MoM surprises from mid-1998 onwards till 2023 in figure 3. First, we want to highlight how massive the forecast errors have been in the recent few years. The average CPI MoM and CPI Core MoM is 0.2%. whereas, in the recent few years, surprises shot up as high as 0.6% for CPI MoM and 0.8% for CPI Core MoM, three and four times the average value.

[2] using the Bloomberg variable Conventional CTD Forward Risk
[3] https://www.federalreserve.gov/pubs/feds/2006/200628/200628abs.html
[4] https://www.federalreserve.gov/data/tips-yield-curve-and-inflation-compensation.htm

Figure 3: **Time Series of CPI Surprises**

(a) CPI MoM

(b) CPI Core MoM

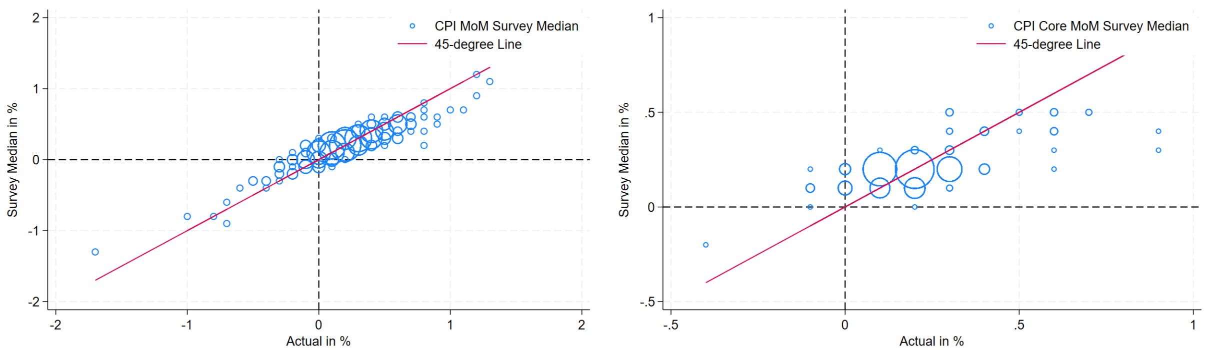


This figure plots the time series of $surprise_t^{CPI}$ which is the difference of the CPI actual or announced value in each month and the CPI Bloomberg Survey Median (in percent) from 1998m6 to 2023m12.

Figure 4: **Actual CPI MoM versus Survey Median**

(a) CPI MoM

(b) CPI Core MoM



This figure plots a scatter plot of the CPI actual or announced value in the x axis and the CPI Bloomberg Survey Median on the y axis from 1998m6 to 2023m12. The red line is a 45 degree line.

We also want to highlight that there were both negative and positive surprises throughout the sample, largely between -.2% and .2%. It shows that the survey participants did not systematically over or under-forecast in particular time periods. To add more validation to the forecast capabilities of the surveyors, we plot a scatter plot of the survey median versus the announced value in Figure 4. Clearly, the forecasts predict the actual values very well.

We now proceed to try and understand the impact of these surprises on various assets. We focus on CPI Core MoM to avoid the volatility driven by food and energy prices. Specifically, we estimate

$$\Delta Y_t = \alpha + \beta \times surprise_t^{CPICoreMoM} + \epsilon_t \tag{13}$$

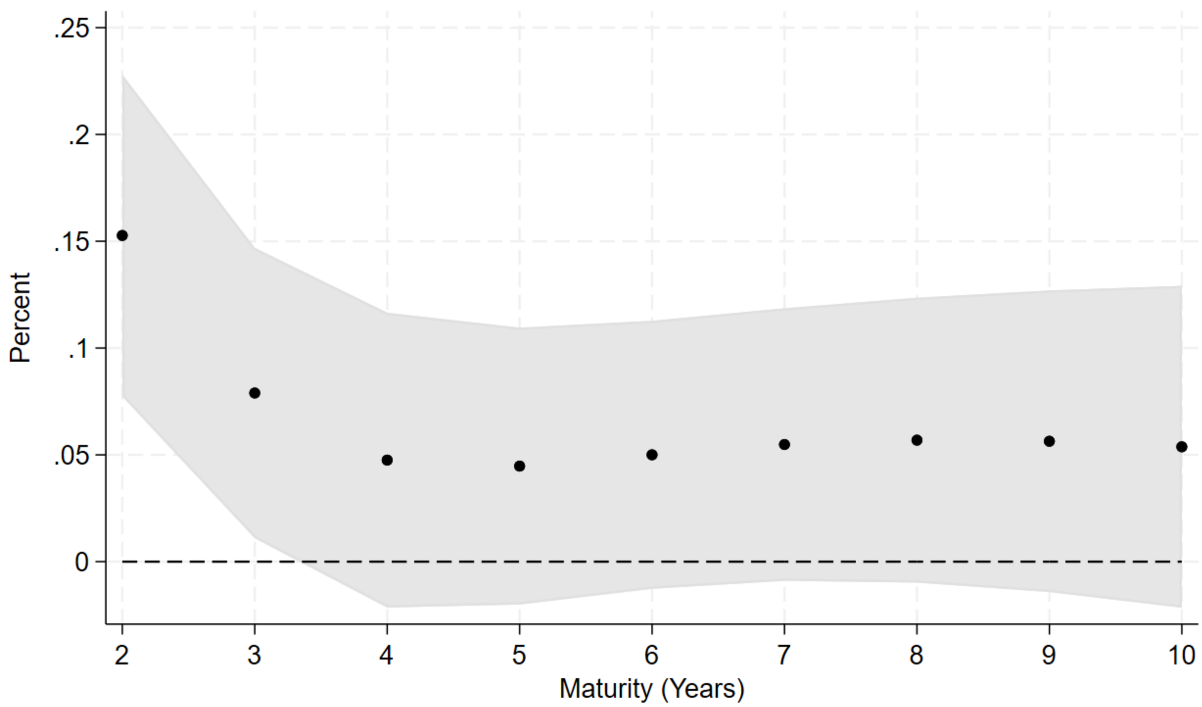
Here ΔY_t is the change in the outcome variable of interest (e.g., forward breakeven inflation rate, he yield on a zero-coupon Treasury bond, log price of S&P500 etc.), $surprise_t^{CPICoreMoM}$ is a measure of the surprise in inflation revealed in the CPI announcement, ϵ_t is an error term, and α and β are regression parameters. The parameter of interest is β, which measures the effect of the surprise in inflation on the asset prices. The CPI Core MoM surprises are positively autocorrelated up to lag one. Therefore, additionally, to check the robustness of the β coefficient, we include a surprise lag as a control and find that the results hardly change. So we stick to the specification without the control.

We first document some facts about the response of future inflation and the bond market to a surprise in inflation. We focus on the years 2004-2023 instead of 1998 onward because announcements became regularly scheduled from 2004 onward, and the TIPS bond market was more illiquid earlier (D'Amico et al., 2018).

Fact 1: Forward breakeven inflation rates increase with a positive surprise in inflation.

First, we look at daily changes in forward breakeven inflation rates as implied by TIPS with maturities from two to ten years (Figure 5). We find that a 1% surprise increase in CPI Core MoM in the past month leads to an average 15 basis point increase in the expected inflation two years ahead. The positive effect on future inflation is muted for longer maturities. This is the key evidence for assuming that the underlying shocks are persistent in the model. This is in line with what is documented in the literature for older years, for example, as in Faust et al. (2007). Note that the magnitude of the response of forward breakeven inflation rates to a surprise in inflation rate varies over the years (see figure 13).

Figure 5: Forward Breakeven Inflation



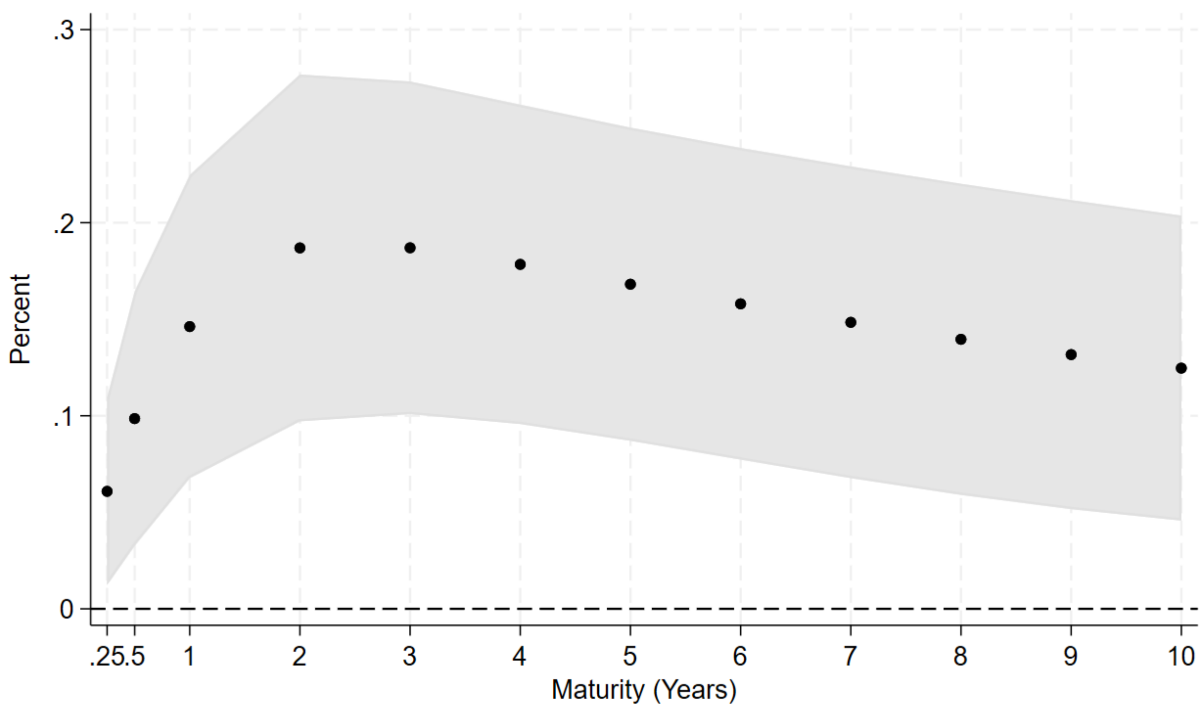
This figure plots the β coefficients (black dots) and their 95% confidence intervals (shaded grey region) for the regression 13 $\Delta Y_t = \alpha + \beta \times surprise_t^{CPICoreMom} + \epsilon_t$ where Δy_t is the daily change in the forward breakeven inflation (in percent) of different maturities and the x axis is the maturity of the Treasury bond in years. t refers to the CPI announcement date. $surprise_t^{CPICoreMoM}$ is defined as in 12 in the text. The time period is from 2004m1-2023m12.

Fact 2: Nominal yields increase with a positive surprise in inflation.

Second, we look at daily changes in nominal yields of zero coupon bonds with maturities from 3 months to ten years (Figure 6). We find that a 1% surprise increase in CPI Core MoM in the past month leads to an average 20 basis point increase in nominal yields maturing two years from now. The positive effect on nominal yields is significantly positive but muted for longer maturities. Thus, when there is a positive surprise in inflation, interest rates in the future years are expected to rise. This is unsurprising if the market expects the Fed to tackle inflation by increasing interest rates. If inflation is expected to persist (as we show in Figure 5), then future interest rates will be expected to rise too. This is one of the reasons for assuming a Taylor rule in our model, apart from it being a standard assumption in New Keynesian models anyway. This is also in line with what is documented in the literature for older years, for example, as in Faust et al. (2007).

One thing to note is that there is some heterogeneity in the response of nominal yields to a surprise in inflation over the years (see figure 7). In recent years, a 1% positive surprise in CPI Core MoM of the past month leads to as high as a 70 basis point response in 2-year ahead nominal yields. Also, as expected, during the zero lower bound periods after the Great Recession and at the onset of COVID-19, nominal yields hardly moved in response to a surprise in inflation. This is further evidence of the fact that these yield responses are driven by the Fed's expected response.

Figure 6: Nominal Yields



This figure plots the β coefficients (black dots) and their 95% confidence intervals (shaded grey region) for the regression 13 $\Delta Y_t = \alpha + \beta \times surprise_t^{CPICoreMom} + \epsilon_t$ where Δy_t is the daily change in the Treasury nominal yields (in percent) of different maturities and the x axis is the maturity of the Treasury bond in years. t refers to the CPI announcement date. $surprise_t^{CPICoreMoM}$ is defined as in 12 in the text. The time period is from 2004m1-2023m12.

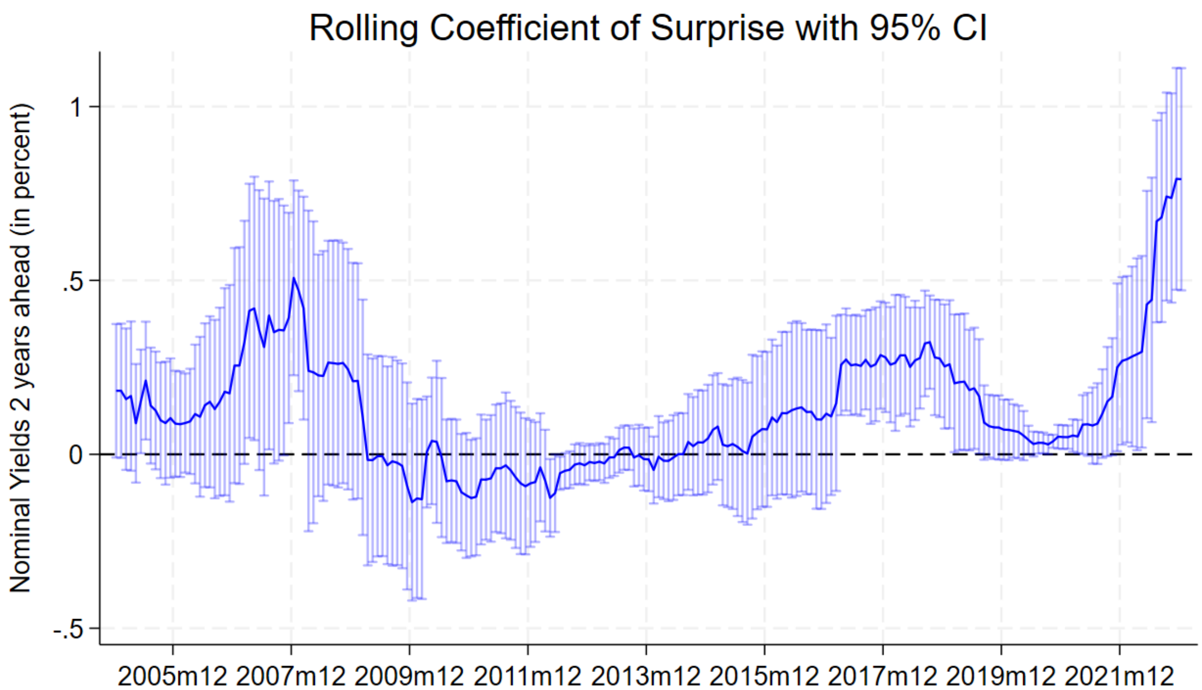
One potential concern with these daily regressions could be that there could be additional information released during the announcement days that are driving these results. To attenuate these concerns, we also construct higher-frequency changes in nominal yields of treasury bonds using Treasury futures data. The CPI announcement occurs at 8:30 AM, and we look at futures around a 30-minute window between 8:20 AM and 8:50 AM. We have treasury futures data for bonds maturing in 2, 5, and 10 years. As shown in Figure 14, the β coefficients for high-frequency change in nominal yields and the daily change in nominal yields are very similar, but as expected, the 95% confidence interval is tighter for the high-frequency coefficients.

An additional concern with the nominal yield regression and the interest rate interpretation could be that most of the movement is driven by changes in risk premia around these announcements. To attenuate these concerns, we use the Kim and Wright (2005)) measure of nominal yields that account for term premia in the bond pricing (refer to Figure 15). Although the positive effect on yield is more muted, the results are still significant. Also, table 3 suggests that the impact of CPI surprises on the volatility index of the stock market is insignificant, further mitigating concerns of risk-premia primarily driving the asset prices.

Fact 3: In response to a positive surprise in inflation, real dividends are expected to rise from 2016-2019 and fall from 2020-2023.

We now examine the stock market's response to an unexpected rise in inflation, as illustrated in Figure 8. Our analysis focuses on S&P 500 dividend futures alongside the index's stock price. Dividend futures are cash-settled derivative contracts available from November 2015 onwards that enable investors to speculate on the future dividend payments of an index. For example, S&P 500 dividend futures for 2025 reflect market expectations regarding the annual dividends of S&P 500 companies at the end of that year. The scatter plot reveals that neither the S&P 500 stock price nor the implied annual dividends, as indicated by dividend futures, exhibit a consistent increase or decrease on average in response to CPI surprises. This is also in line with the literature that suggest that the response of the stock market to macroeconomic news releases in state-dependent (McQueen and Roley, 1993). A potential concern with looking at the dividend futures response is whether it captures something besides the future dividend expectations for S&P 500 companies. Since these contracts are settled in December 2025 based on actual dividends, their pricing should not be directly affected by prevailing interest rates. However, dividend futures could be affected by risk premia. Provided we assume that risk premia remains relatively stable around CPI announcements (refer to Table 3), we can reasonably interpret dividend futures as reflecting

Figure 7: Response of nominal yield expectations to surprise in CPI over the years



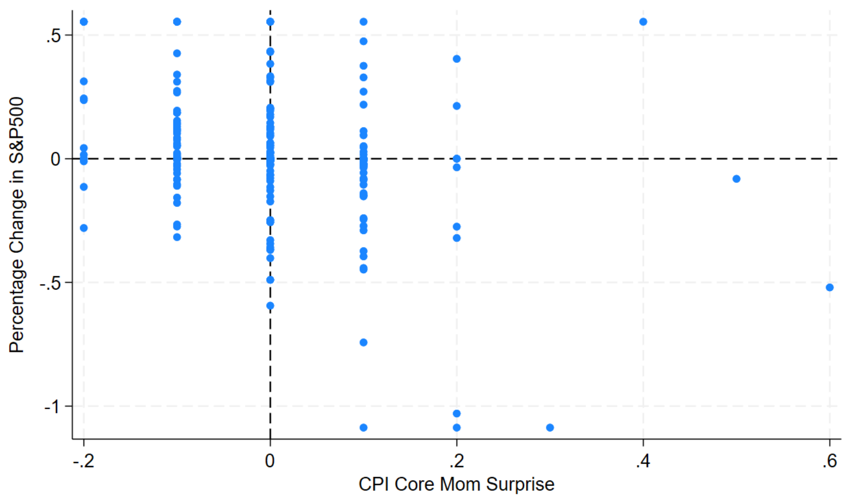
This figure plots the rolling β coefficients (blue line) and their 95% confidence intervals (blue bars) for the regression 13 $\Delta Y_t = \alpha + \beta \times surprise_t^{CPICoreMom} + \epsilon_t$ where Δy_t is the daily change in the two year ahead nominal yields (in percent) and t refers to the CPI announcement date. The x axis is the month of the announcement. The window size is 24 observations i.e. ± 1 year around the month of observation in the x axis. $surprise_t^{CPICoreMoM}$ is defined as in 12 in the text.

market expectations of future dividend payouts.

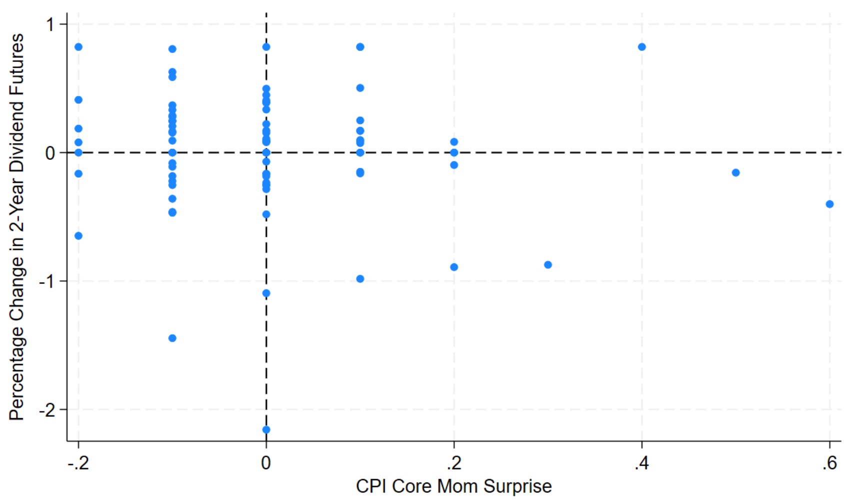
Now, we try to understand if the response of dividends exhibits any consistent pattern over some time periods. We focus on the response to real dividends varies over the years (see Figure 9). We construct a measure of change in expectations of real dividends 2 years ahead around the announcements. We focus on 2 years ahead because the earliest measure of breakeven inflation using TIPS is 2 years ahead. We construct the real dividend measure as follows. Expectations of nominal dividends two years ahead are assumed to be given by the average of nominal dividend futures that are settled in December of (a) the next year and (b) the year after that. So for example, if CPI announcement occurs in May 2020, dividends futures that are settled in December 2021, and December 2022 are looked at, and their average is taken as the two year ahead (from the announcement date) expectations of nominal dividends. We subtract these expectations with 2 times the 2 year ahead annual breakeven inflation rate as measured from TIPS, to get daily expectations of real dividends two years ahead. We, thus, study changes in daily expectations of real dividends two years ahead around announcements, using changes in nominal dividend futures and breakeven inflation rates. We find that real dividends two years ahead were expected to increase in response to a positive surprise in inflation in the years 2016-2019 but decrease in 2020-2023. This will be used to construct expectations of future consumption in the later sections and we will show that this change in the response is going to be useful to show the increasing strength of supply shocks in inflation post-Covid.

Figure 8: **Stock Market and CPI Surprises**

(a) S&P500

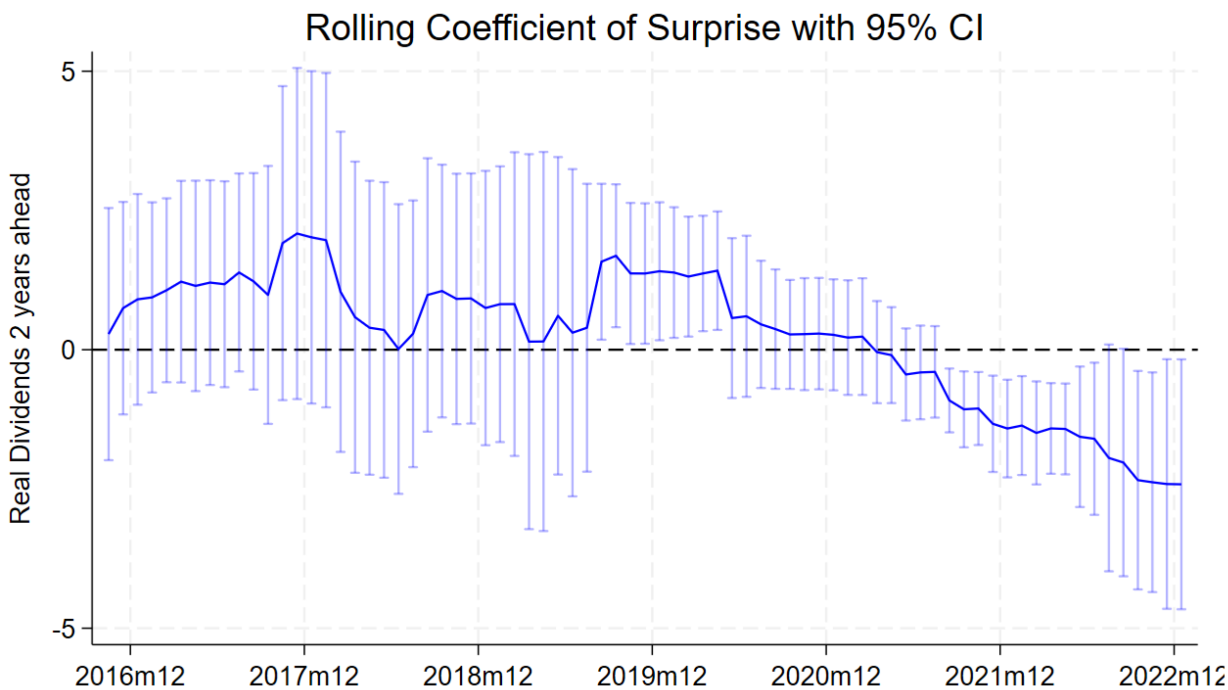


(b) Dividends



This figure plots a scatter plot of the CPI Core Mom Surprise on the x axis as defined by 12 in the text. The y axis plots (a) the percentage change in price (open-close) of S&P 500 and (b) the daily percentage change in price of Dividend futures of S&P 500 expiring in 2 years. The percentage change in price is winsorized at 5% from the top and 1% from the bottom. The CPI announcement dates around which the Surprise and daily percentage change in price is calculated are from 2004m1-2023m12 for S&P 500 stock price and from 2015m11-2023m12 for dividend futures.

Figure 9: Response of real dividend expectations to surprise in CPI over the years



This figure plots the rolling β coefficients (blue line) and their 95% confidence intervals (blue bars) for the regression 13 $\Delta Y_t = \alpha + \beta \times surprise_t^{CPICoreMom} + \epsilon_t$ where Δy_t is the percentage change in 2 year ahead real dividend expectations around the CPI announcement and t refers to the CPI announcement date. The x axis is the month of the announcement. The window size is 24 observations i.e. $\pm$ 1 year around the month of observation in the x axis. $surprise_t^{CPICoreMoM}$ is defined as in 12 in the text.

To summarize, we find that nominal yields and forward breakeven yields respond positively, significantly, and persistently to a positive surprise in inflation. The stock market response varies over the years. Particularly, two-year ahead real dividends of S&P 500 companies were expected to rise from 2016-2019 but expected to fall from 2020-2023. In the next section, we use this reduced-form evidence in our model.

4 Matching the Model to the Data

In this section, we use a combination of the model and data to infer how the market decomposed unexpected inflation into demand and supply components over the years.

4.1 Constructing observables that map to the model

The revision of beliefs given by (11) of each of the underlying shocks is unobservable in the data. However, we can extract these beliefs by observing asset price movements around announcements.

Since consumption and nominal interest rate are linear combinations of the underlying shocks (bBy (10) and (8)), the revision of beliefs about them around announcement after π_t is announced as say $\bar{\pi}_t$ can be given by

$$E_{ht}\left[i_t | \pi_t = \bar{\pi}_t\right] - E_{ht}^{ba}\left[i_t\right] = \Sigma_k \underbrace{a_i^k}_{\text{weight of } k \text{ in } i} \underbrace{\frac{a_\pi^k \sigma_h^{2k}}{\sigma_h^{2\pi}}}_{\text{revision of shock } k} \underbrace{(\bar{\pi}_t - \mu_{ht}^\pi)}_{surprise_{ht}^\pi} \tag{14}$$

The left-hand side is changes in expectations of nominal interest rates and can be inferred from the changes in treasury yields around CPI announcements. The assumption here is that the risk-premium associated with these asset prices does not change around the announcement. This is somewhat supported by table 3, which shows that the VIX does not move significantly in response to CPI surprises. The surprise in inflation can be inferred from the Bloomberg surprise in CPI as defined in 12. The green coefficient is the weight of shock k in interest rate i times the update in belief of shock k as derived in proposition 1, summed over the two shocks of demand and supply. Also, since $a_i^k, a_\pi^k > 0$ for all k, i.e. interest rates and inflation increases with positive demand shock and negative supply, the green coefficient must be positive. Thus, when there is a positive surprise in inflation, both the positive demand shock and the negative supply shock are positively revised. The green coefficient can simply be matched to the OLS regression coefficient when changes in nominal yields are regressed on the surprise in CPI. As verified by the Figure 7, the response is positive as predicted by the model, with the exception of the zero lower bound period.

The changes in output expectations are given by

$$E_{ht}\left[c_t|\pi_t = \bar{\pi}_t\right] - E_{ht}^{ba}\left[y_t\right] = \Sigma_k \underbrace{a_c^k}_{\text{weight of } k \text{ in } c} \underbrace{\frac{a_\pi^k \sigma_h^{2k}}{\sigma_h^{2\pi}}}_{\text{revision of shock } k} \underbrace{(\bar{\pi}_t - \mu_{ht}^\pi)}_{surprise_{ht}^\pi} \tag{15}$$

The left hand side measure changes in consumption expectations around announcement. It is hard to find a direct counterpart to consumption expectations in the assets market. Theoretically, the finance literature has assumed that aggregate real dividends are proportional to aggregate consumption, i.e., $c_t = \kappa d_t$ in many models (for example, Campbell (2003)). We use real annual dividend futures of S&P 500 (elaborated in section 3.2) to measure changes in consumption expectations by constructing a tracking portfolio (Lamont, 2001). To recap, dividend futures are cash-settled derivative contracts that allow investors to speculate on the future dividend payments of an index. For example, dividend futures of S&P 500 index for the year 2025 represent the market speculation of what the annual dividends of S&P 500 companies are going to be at the end of the year 2025. Depending on what the actual dividends turn out to be, the trade gets settled in December 2025. Thus, dividend futures do not need to be discounted by the prevailing interest rates. However, the dividend futures might be contaminated by risk premia. As long as we believe that risk premia does not change much around CPI announcements (see table 3), we can assume that dividend futures represent market expectations of future dividend payouts.

An alternate measure of consumption expectations would be to use the stock price change of S&P 500, following papers such as Campbell et al. (2020), Cieslak and Pflueger (2023), Jarociński and Karadi (2020) and Pflueger (2023). The implicit assumption, however, is that stocks are a levered claim on *consumption* following Abel (1990). Dividend futures of S&P 500, on the other hand, might be a more relevant measure of consumption expectations as it refers to the cashflow component of these stocks and is not contaminated by yield curve changes (Nagel and Xu, 2024). Apart from that, one could directly use changes in expectations of firm profits in the New Keynesian model instead of changes in consumption expectations. However, generating meaningful procyclicality of firm profits in a New Keynesian model requires a considerable amount of additional modeling structure (see Bilbiie and Känzig (2023)).

The way we construct a tracking portfolio for consumption using dividend futures is as follows. We measure the actual monthly nominal consumption growth of non-durables and services using data from NIPA. We measure monthly growth in expectations of total nominal dividend payouts of S&P 500 companies at the end of the year, i.e. in December, from dividends futures. One caveat is that dividend futures are only available November 2015 onwards and we use the data until December 2023 . At the monthly level, we regress nominal consumption growth on the growth in expectations of nominal dividends end of the year controlling for 2 yr, 5 yr, and 10 yr breakeven inflation rates (as measured from TIPS). Thus we control for inflation, to interpret the relationship between the two variables to be real instead of nominal. We find a highly significant regression coefficient of 0.26 and an R-squared of 30%. Thus, if there is a 1% month-over-month growth in expectations of real dividends at the end of the year, it leads to a 0.26% growth in actual real consumption month over month. This coefficient is quite robust to controlling for a wide number of variables like nominal yields, breakeven inflation rates and dividend futures for later years (see Table 1). Thus, assuming the relationship $c_t = \kappa d_t$, we set $\kappa = 0.26$.

We construct a measure of change in expectations of real dividends 2 years ahead around the announcements. We focus on 2 years ahead because the earliest measure of breakeven inflation using TIPS is 2 years ahead. Another reason is that we want to focus on more medium-term dynamics than short-term dynamics which could be driven by a lot of other temporary factors. As mentioned in section 3.2, we construct the two year ahead real dividend measure using the dividend futures and breakeven inflation rates. By the regression analysis in the previous paragraph, the daily changes in real consumption expectations 2 years ahead around CPI announcements would be 0.26 times the changes in daily expectations of real dividends two years ahead. Thus, the left-hand side part of equation 15 can now be measured using asset prices.

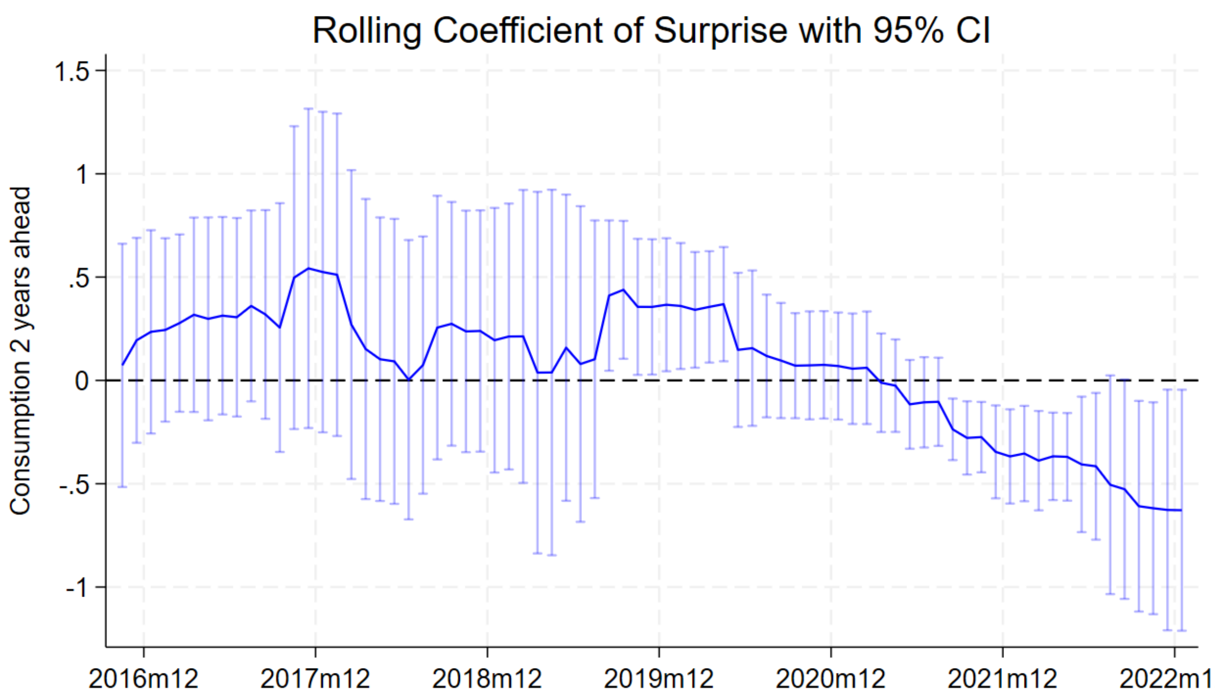
Table 1: Tracking Portfolio for Consumption

	(1)	(2)
Dividend (same year)	**0.24*****	**0.26*****
	(0.06)	(0.09)
Dividend (one year ahead)		-0.07
		(0.09)
Dividend (two years ahead)		-0.05
		(0.10)
Dividend (three years ahead)		-0.00
		(0.12)
Dividend (four years ahead)		0.08
		(0.06)
Breakeven inflation (two years ahead)		0.41*
		(0.24)
Breakeven inflation (five - two years ahead)		1.19
		(0.95)
Breakeven inflation (ten - two years ahead)		-0.43
		(0.93)
Nominal Yields (two years ahead)		-0.17*
		(0.08)
Nominal Yields (five - two years ahead)		-0.79
		(0.58)
Nominal Yields (ten - two years ahead)		0.27
		(0.42)
BAA - AAA bond credit spread		0.22
		(0.18)
Constant	0.41***	-0.27
	(0.04)	(0.45)
Observations	89	89
R^2	0.196	0.424

Results from estimating $\text{Cons}_t = \alpha + \beta \times \text{Div}_t + \delta X_t + \epsilon_t$ where $Cons$ refers to the nominal consumption growth of non durables and services at month t from November 2015 till December 2023, trimmed top and bottom at 5%. The regressors are Div_t which is the month t expectations of total nominal dividends implied by the end of the year dividend futures of S&P 500 companies. The controls X_t include dividend futures for one year to four years ahead, and two year, five year and ten year ahead treasury nominal yields and breakeven inflation rates averaged in month t.

In the right hand side, the green term can either be positive or negative since $a_c^d > 0, a_c^s < 0$, i.e., consumption increases with a positive demand shock but decreases with a negative supply shock. Figure 10 shows that the response of future expected consumption was positive before COVID-19 but became negative after COVID-19, highlighting the increasing role of supply shocks post COVID-19.

Figure 10: Response of consumption expectations to surprise in CPI over the years



This figure plots the rolling β coefficients (blue line) and their 95% confidence intervals (blue bars) for the regression 13 $\Delta Y_t = \alpha + \beta \times surprise_t^{CPICoreMom} + \epsilon_t$ where Δy_t is the percentage change in 2 year ahead real consumption expectations around the CPI announcement and t refers to the CPI announcement date. The x axis is the month of the announcement. The window size is 24 observations i.e. $\pm$ 1 year around the month of observation in the x axis. $surprise_t^{CPICoreMoM}$ is defined as in 12 in the text.

Table 2: Parameter values taken from the Literature

Parameter	Definition	Value
θ	Calvo	0.9
β	Discount factor	0.995
γ	CRRA	1
ψ	Frisch elasticity	1
ϕ_π	Taylor rule	1.5
ϕ_y	Taylor rule	0.5
ρ_s	Persistence of shock	0.9
ρ_d	Persistence of shock	0.9
f_s	Relative firm signal precision	1
f_d	Relative firm signal precision	0.8

4.2 Findings

Throughout this section, some of the parameters of the model will be kept fixed. These are the discount rate β, the inverse of the intertemporal elasticity of consumption γ, the curvature of the disutility of supplying labour, the parameters in the Taylor-type rule ϕ and the persistence of the structural shock ρ. These will be set as $\{\theta, \beta, \gamma, \varphi, \phi_\pi, \phi_y, \rho_d, \rho_s\} = \{0.9, 0.995, 1, 1, 1.5, 0.5, 0.9, 0.9\}$ (see table 2).

The parameterisation of the discount factor β at 0.995 reflects that a period in the model should be interpreted as being one month, to match with the frequency of announcements. The Calvo parameter θ should thus be interpreted as the fraction of firms that do not change prices in a given month and it will be set to $\theta = 0.9$ which implies an average price duration of 10 months. The choice of the exogenous persistence parameter for labour supply shock $\rho_s = 0.9$ roughly reflects the persistence of various measures of marginal cost (for instance the labour share in GDP) and is also used in Nimark (2008) where each time-period is also a month. We set the same value for persistence of demand shock ρ_d. Finally, The precision ratio of the signals of the shocks received by firms given by (f_d, f_s) is set to $(0.8, 1)$. $f_s = 1$ means firms know the supply shock with certainty, whereas $f_d < 1$ shows that they have dispersed information about the demand shock and do not know it with certainty. $f_d < f_s$ is to capture the fact that the price-setting firms have more precise information about the supply conditions rather than demand conditions. It does not matter for the qualitative results and only what f_d and f_S are relative to each other that matter for quantitative results.

With these parameters, we can recover $\boldsymbol{a_c}, \boldsymbol{a_\pi}, \boldsymbol{a_i}$ in 8, 9 and 10. We define the market-perceived share of demand in unexpected inflation as:

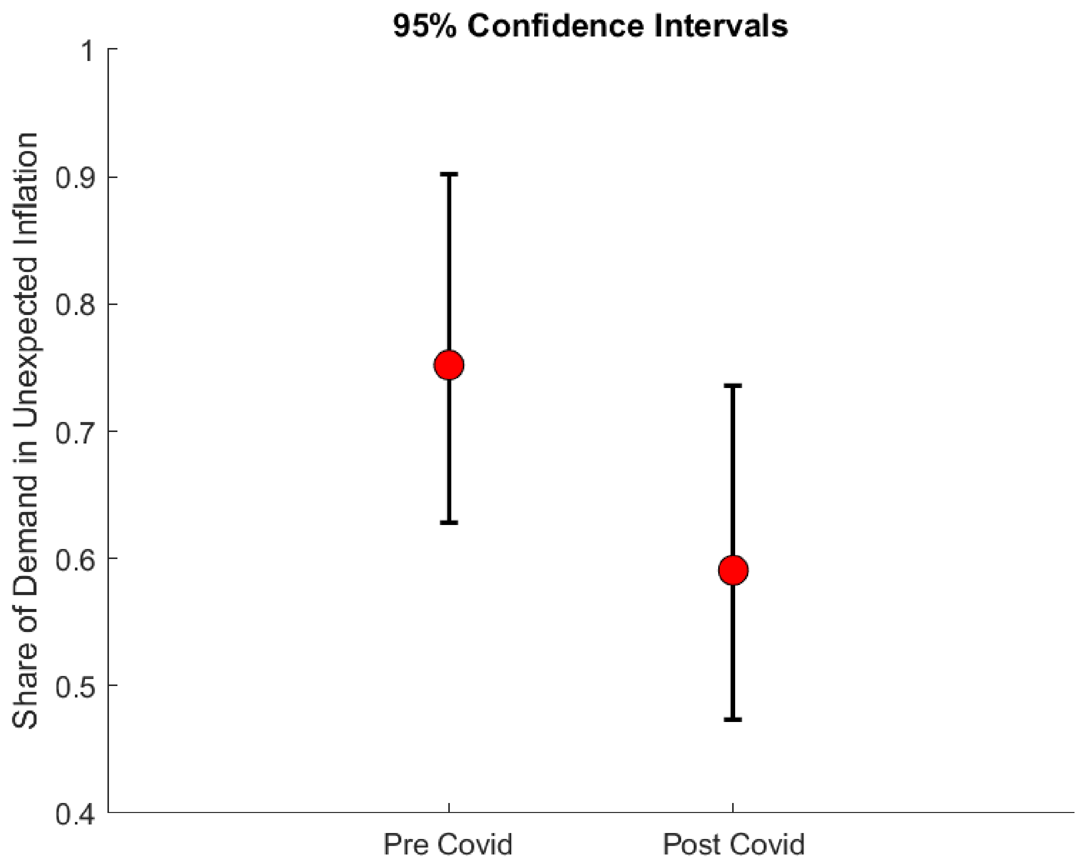
$$\begin{aligned}\text{Share}_{dd} &= \frac{(a_\pi^d)^2\sigma_h^{2d}}{(a_\pi^d)^2\sigma_h^{2d} + (a_\pi^s)^2\sigma_h^{2s}} \\ &= \frac{(a_\pi^d)^2 V_d}{(a_\pi^d)^2 V_d + (a_\pi^s)^2 V_s}\end{aligned} \tag{16}$$

where $V_k = \sigma_h^{2k}/\sigma_h^{2\pi}$ for $k \in (d, s)$ is used for normalization. Now, V_d, V_s can be recovered from 14 and 15 (where the revision of each shock is multiplied by ρ_k^{24} to adjust for 2 year ahead asset prices) and the $\boldsymbol{a}$ is given by the calibration of the New Keynesian model parameters. After the matching exercise and some further algebra (see A.2), we can show that

$$\text{Share}_{dd} = \frac{\frac{a_\pi^d}{\rho_d^{24}}(a_c^s - a_i^s \frac{\Delta c_{2yr}^{obs}}{\Delta i_{2yr}^{obs}})}{\frac{a_\pi^d}{\rho_d^{24}}(a_c^s - a_i^s \frac{\Delta c_{2yr}^{obs}}{\Delta i_{2yr}^{obs}}) + \frac{a_\pi^s}{\rho_s^{24}}(-a_c^d + a_i^d \frac{\Delta c_{2yr}^{obs}}{\Delta i_{2yr}^{obs}})} \tag{17}$$

where $\frac{\Delta c_{2yr}^{obs}}{\Delta i_{2yr}^{obs}}$ is the change in 2 year ahead consumption expectations around CPI announcement divided by the change in 2 year ahead interest rate expectations around announcement. ρ_d^{24} and ρ_s^{24} occurs because the model is at monthly frequency, hence the auto-correlation coefficient is raised to the power 24 to account for two year ahead expectations. Thus, we have a sufficient statistic for the share of demand in unexpected inflation that only depends on $\boldsymbol{a}$, ρ and the *ratio* of change in consumption expectations around announcement divided by the change in interest rate expectations around announcement $\frac{\Delta c_{2yr}^{obs}}{\Delta i_{2yr}^{obs}}$. It is also independent of the level of surprise. To understand why, suppose the change in observed asset prices representing interest rates Δi_{2yr}^{obs} and consumption Δc_{2yr}^{obs} doubles, then by 14 and 15, V_d and V_s would also double, but the share of demand in unexpected inflation Share_{dd} would be multiplied by 2 in both numerator and denominator, thus remaining unchanged. Thus, share of demand only depends on the ratio and it depends *positively*, i.e, more positive the change in the consumption change to interest rate change ratio, higher the share of demand in unexpected inflation. This is intuitive, because a positive demand shock leads to a positive suprise in inflation and increases consumption expectations, while a negative supply shock leads to a positive surprise in inflation but decreases consumption expectations.

Figure 11: Share of demand shock in variance of inflation over the years

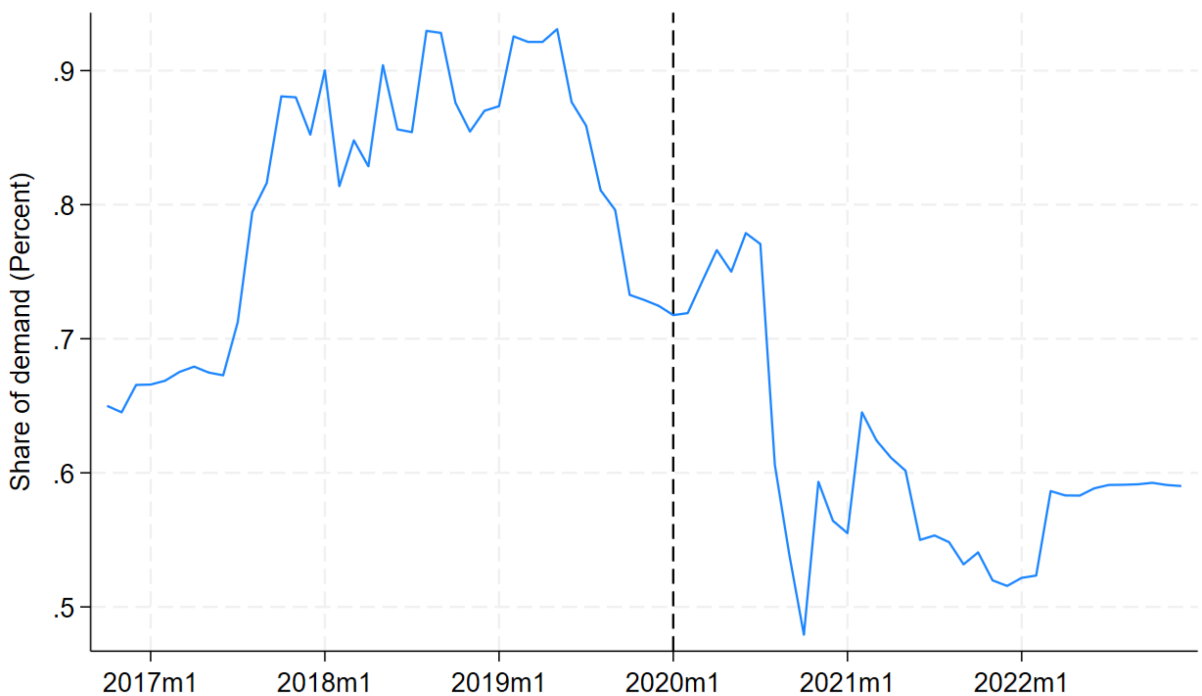


This figure plots the share of demand shock in variance of inflation over the years that is calculated by the method described in the section 4.2.

First, We do a regression of Δc_{2yr}^{obs} on Δi_{2yr}^{obs} for the years before Covid (November 2015 - December 2019) and after Covid (January 2020 - December 2023) separately and construct the share of demand shock (see Figure 11). Before COVID-19, the share of demand in unexpected inflation was around 80%, whereas it was around 60% post COVID-19. Thus, the share of supply increased by 20 percentage points post covid. This is not surprising if we look at the response of consumption expectations to a surprise in inflation over the years (see Figure 10). Consumption expectations increased with a positive surprise in inflation before covid, while the response turned negative around 2021 onwards, indicative of the increasing role of supply shocks.

We get similar results if we do a rolling regression over the years instead of two sub-periods. We do a rolling regression of Δc_{2yr}^{obs} on Δi_{2yr}^{obs} with a window size of 24 observations (or 2 years) over the years and construct the share of demand shock (see Figure 12). To be clear, the 2020m1 in the x-axis represents a the regression coefficient of Δc_{2yr}^{obs} on Δi_{2yr}^{obs} for the years 2019m1-2021m1. Again, as expected before 2020, the share of demand in unexpected inflation was around 80%, whereas it was around 60% 2020 onwards on an average. There is a slight increase in the demand share after 2020m1. This could be because of the unprecedented fiscal transfers that occurred in 2020 or because supply chain shortages in some sectors at the start of the pandemic could be picked up as a demand shock if interest rates are at a zero lower bound (Guerrieri et al., 2022).

Figure 12: Share of demand shock in variance of inflation over the years



This figure plots the share of demand shock in variance of inflation over the years that is calculated by the rolling regression method described in the section 4.2.

Our analysis shows that supply chain uncertainties have been perceived to play a major role in inflation since 2020. This information becomes relevant for central bankers who claim to respond differently to demand versus supply shocks. It also becomes especially relevant to plan for all possible future scenarios and for an eventual soft or hard landing by the Fed, as the probability of large negative supply shocks in the future increases with higher uncertainty.

4.3 Sensitivity of the Share in Unexpected Inflation to Parameter Values

In Figure 16, we explore how sensitive the share of demand in unexpected inflation is to different parameter values. The share curve largely maintains its shape for different parameter values of the relative information precision of the signal of the firms (f_d, f_s), the Taylor coefficients ($\phi_y^{Tay}, \phi_\pi^{Tay}$). the inverse of the intertemporal elasticity of substitution of income (γ) and the Frisch elasticity of labor supply (ψ). Thus, our key result that the share of unexpected demand in inflation falls by 20 percentage points post covid doesn't change much. The shape does vary slightly with the autocorrelation coefficients ρ_d, ρ_s, but this is because they not only affect the **a**, i.e., the weight of each of the underlying shock to macro variables of interest rate, inflation and output, but also have a direct effect (see equation A.2). This is because how much the two year ahead interest rate or consumption expectations should move directly depends on the persistence of the underlying shocks. However, the share of demand shock in unexpected inflation definitely fell post-covid.

4.4 Role of Monetary Policy in this model

The interpretation of monetary policy in this model depends on the information set of the firms vis-à-vis the household. If firms and household have the same level of information about the monetary shock, that is, the error term in the Taylor rule, then there will be no belief update by the household about the monetary shock when the inflation announcement occurs. This could happen when there are only public signals about the monetary shock, and no private information. FOMC announcements could be such an example of public signals where the communication is from the central banks to all the agents in the economy. Thus, for the purposes of our question, this would be equivalent to assuming there is no error term in the Taylor rule, which is what we do.

If firms have private information about the monetary policy shocks beyond what the household knows, then household will also update their belief about monetary shocks from the inflation announcement. In such a model, an expansionary monetary shock will act very similar to a positive demand shock. Thus demand shocks will absorb the role of a monetary policy shock in such a situation. If firms have incorrect information about the monetary policy rule, then the model becomes trickier to solve. An example could be that the Taylor coefficients are time-varying or asymmetric, and firms have incorrect perceptions about them. Supply shocks might absorb some role of monetary policy in those situations.

5 Conclusion

We use a standard New Keynesian model with incomplete information and and inflation announcement to interpret asset price movements around CPI announcements. We find that the share of supply in unexpected inflation has increased by 20 percentage points post Covid.

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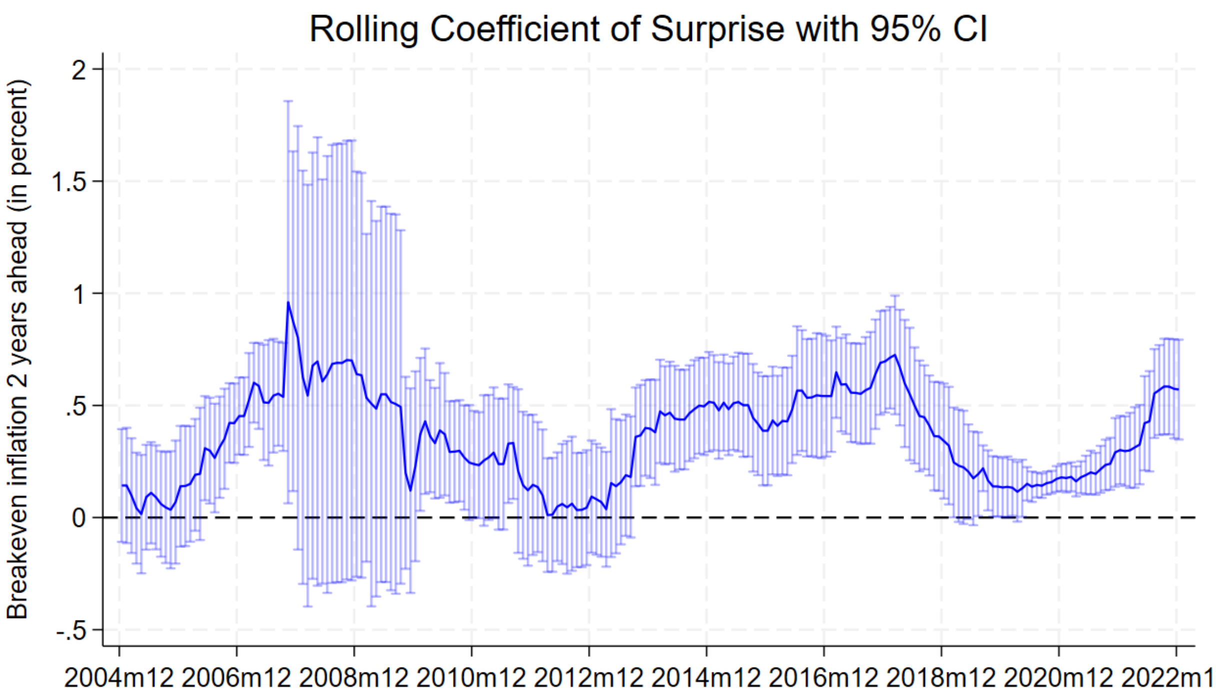
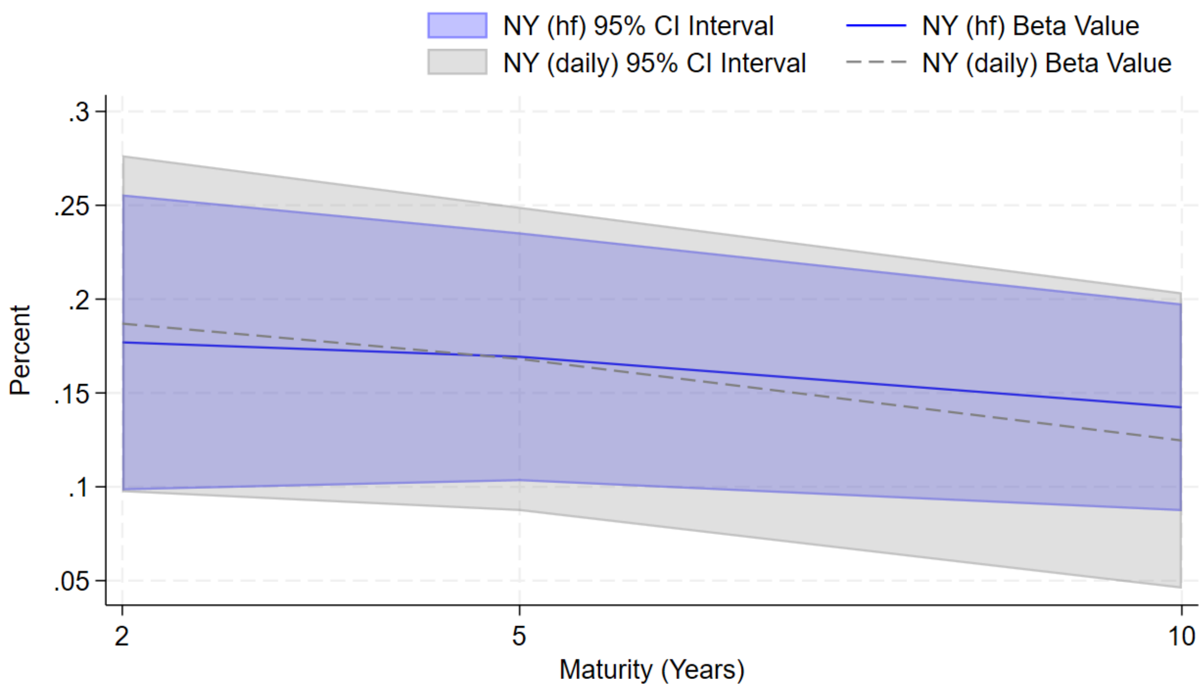
6 Figures

Figure 13: Response of breakeven inflation to surprise in CPI over the years



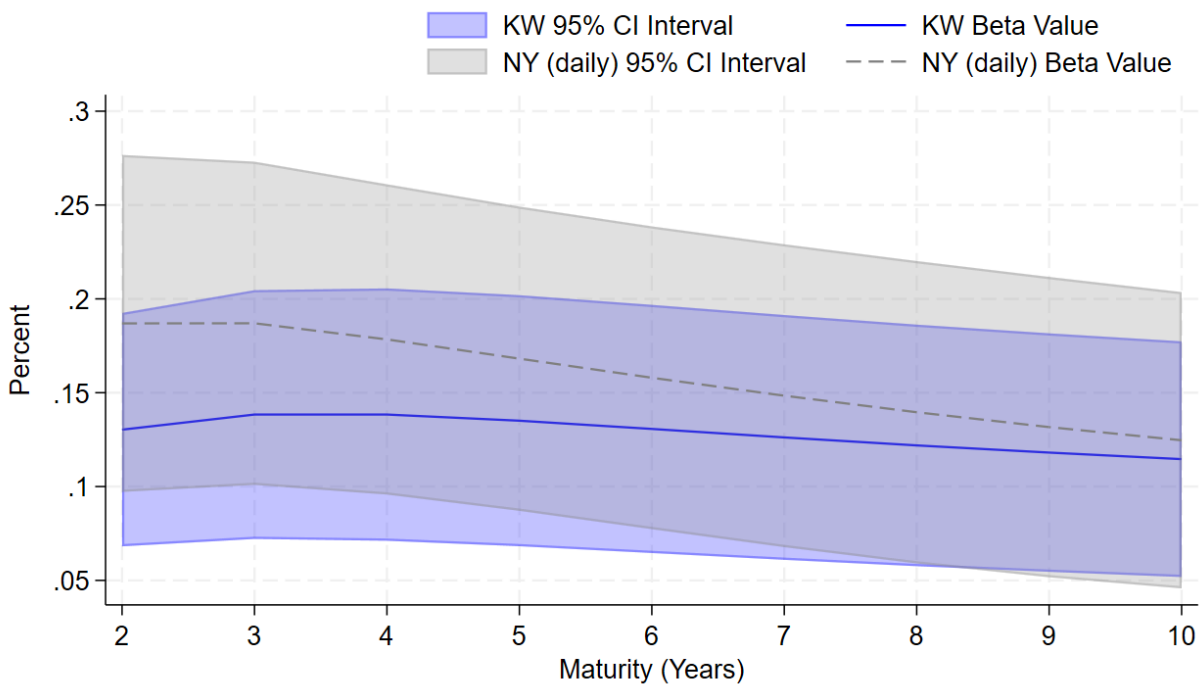
This figure plots the rolling β coefficients (blue line) and their 95% confidence intervals (blue bars) for the regression 13 $\Delta Y_t = \alpha + \beta \times surprise_t^{CPICoreMom} + \epsilon_t$ where Δy_t is the daily change in the two year ahead breakeven inflation (in percent) and t refers to the CPI announcement date. The x axis is the month of the announcement. The window size is 24 observations i.e. $\pm$ 1 year around the month of observation in the x axis. $surprise_t^{CPICoreMoM}$ is defined as in 12 in the text.

Figure 14: Nominal Yields (high frequency)



This figure plots the β coefficients (line) and their 95% confidence intervals (shaded region) for the regression 13 $\Delta y_t = \alpha + \beta \times surprise_t^{CPI} + \epsilon_t$ where x axis is the maturity of the Treasury bond in years. The red and blue color refers to when Δy_t is the daily and the high-frequency change in Treasury nominal yields (in percent) respectively. t refers to the CPI announcement date. $surprise_t^{CPICoreMoM}$ is defined as in 12 in the text. The time period is from 2004m1-2023m12.

Figure 15: Nominal Yields (accounting for risk premia)



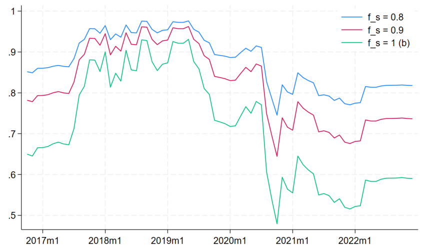
This figure plots the β coefficients (line) and their 95% confidence intervals (shaded region) for the regression 13 $\Delta y_t = \alpha + \beta \times surprise_t^{CPI} + \epsilon_t$ where x axis is the maturity of the Treasury bond in years. The red color refers to when Δy_t is the daily change in Treasury nominal yields (in percent). The blue color refers to when Δy_t is the Kim and Wright (cite) measure of change in Treasury nominal yields (in percent) that accounts for term premia. t refers to the CPI announcement date. $surprise_t^{CPICoreMoM}$ is defined as in 12 in the text. The time period is from 2004m1-2023m12.

Table 3: Volatility Index

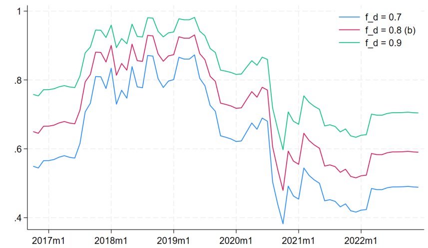
	(1)	(2)
CPI Core MoM Surprise	-0.15	0.45
	(0.34)	(1.59)
Observations	236	236
R^2	0.001	0.001

Results from estimating $\Delta Y_t = \alpha + \beta \times surprise_t^{CPICoreMom} + \epsilon_t$ where column (1) refers to the case where Δy_t is the open price at date t minus the close price at date $t-1$ of the CBOE Volatility Index or VIX and column (2) refers to the case where Δy_t is the close price at date t minus the close price at date $t-1$ of VIX. t refers to the CPI announcement date. $surprise_t^{CPICoreMoM}$ is defined as in 12 in the text. The time period is 2004-2023.

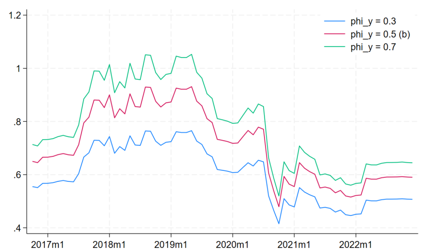
Figure 16: Sensitivity of share of demand in unexpected inflation to different parameter values



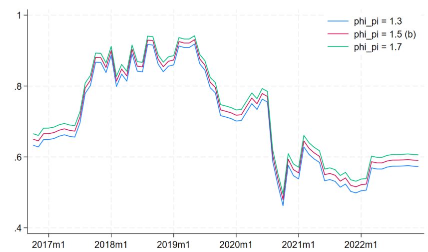
(a) f_s



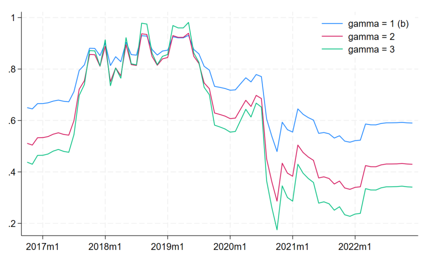
(b) f_d



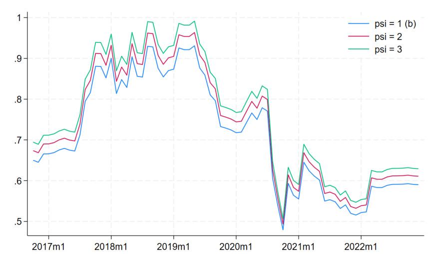
(e) ϕ_y^{Tay}



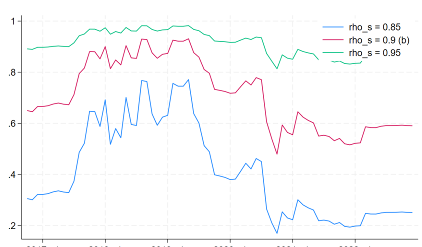
(f) ϕ_π^{Tay}



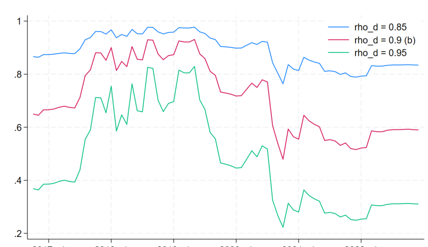
(g) γ



(h) ψ



(c) ρ_s



(d) ρ_d

Each subplot in this figure plots the share of demand shock in the variance of inflation over the years that is calculated by the method described in the section 4.2 but by varying different values of a particular parameter. The bracket (b) in the legend refers to the baseline parameter value chosen.

A APPENDIX

A.1 Proof of Lemma 1 with dispersed information

From Stage 2, the Euler Equation is given by

$$y_t = E_t y_{t+1} - \frac{i_t - E_t \pi_{t+1}}{\gamma} + z_t^d \tag{18}$$

From Stage 2, the Taylor Rule is given by:

$$i_t = \phi_\pi^{Tay} \pi_t + \phi_y^{Tay} (y_t - y_t^n) \tag{19}$$

where $y_t^n = -z_t^s / (\gamma + \psi)$,

In a dispersed information setting, firms j have dispersed signals about the underlying shocks $k \in (dd, ss)$

$$x_{jt}^k = z_t^k + u_{jt}^k$$

where $u_{jt}^k \sim \mathcal{N}(0, \sigma_k^2)$

If firm j is allowed to reset their price, they will choose the optimal price $p_t^*(j)$ that will maximise their profit. Let $\pi_t^*(j) = p_t^*(j) - p_{t-1}$ be called the optimal reset inflation for firm j and is given by

$$\pi_t^*(j) = (1 - \beta\theta) E_{jt} \hat{mc}_t + E_{jt} \pi_t + \beta\theta E_{jt} \pi_{t+1}^*(j) \tag{20}$$

where $\hat{mc}_t = (\gamma + \psi) y_t + z_t^s$. This is exactly how a firm sets its price in a New Keynesian model in Gali (2003), except now it is firm specific expectations E_{jt} instead of E_t.

Now, exante at t, all firms will be identical at $t+1$ because all the shocks are visible to all the firms at the end of the period t. Thus, $\pi_{t+1}^*(j) = p_{t+1}^*(j) - p_t$ and is exante expected to be the same for all j and is equal to reset inflation averaged across all the firms π_{t+1}^*. The average reset inflation is given by

$$\pi_t^* = \int_j \pi_t^*(j) dj \tag{21}$$

Since only $(1 - \theta)$ fraction of randomly chosen firms can choose their price, the aggregate inflation will be $1-\theta$ times average reset price of all firms i.e.,

$$\pi_t = (1-\theta)\pi_t^* \tag{22}$$

Let us assume (ignoring constants)

$$y_t = a_y^d z_t^d + a_y^s z_t^s$$

$$\pi_t = a_\pi^d z_t^d + a_\pi^s z_t^s$$

Rewriting 20, we get

$$\pi_t^*(j) = (1-\beta\theta)E_{jt}\{(\gamma+\psi)y_t + z_t^s\} + E_{jt}\pi_t + \beta\theta E_{jt}\pi_{t+1}^*$$

Integrating over j we get

$$\pi_t^* = (1-\beta\theta)\int_j E_{jt}\{(\gamma+\psi)y_t + z_t^s\}dj + \int_j E_{jt}\pi_t dj + \beta\theta\int_j E_{jt}\pi_{t+1}^* dj$$

Substituting 22

$$\pi_t/(1-\theta) = (1-\beta\theta)\int_j \{E_{jt}(\gamma+\psi)y_t + z_t^s\}dj + \int_j E_{jt}pi_t dj + \beta\theta/(1-\theta)\int_j E_{jt}\pi_{t+1} dj$$

$$\frac{a_\pi^d z_t^d + a_\pi^s z_t^s}{(1-\theta)} = (1-\beta\theta)\int_j E_{jt}\{(\gamma+\psi)(a_y^d z_t^d + a_y^s z_t^s) + z_t^s\}dj + \int_j E_{jt}a_\pi^d z_t^d + a_\pi^s z_t^s dj + \beta\theta/(1-\theta)\int_j E_{jt}a_\pi^d z_{t+1}^d + a_\pi^s z_{t+1}^s dj$$

$$\frac{a_\pi^d z_t^d + a_\pi^s z_t^s}{(1-\theta)} = \int_j E_{jt}z_t^d dj \times [(1-\beta\theta)(\gamma+\psi)a_y^d + a_\pi^d + \beta\theta/(1-\theta)\rho_d a_\pi^d] \tag{23}$$

$$+\int_j E_{jt}z_t^s dj \times [(1-\beta\theta)\{(\gamma+\psi)a_y^s + 1\} + a_\pi^s + \beta\theta/(1-\theta)\rho_s a_\pi^s] \tag{24}$$

Now, $\int_j E_{jt}z_t^k dj = \int_j f_k x_{jt}^k dj + (1-f_k)\rho_k z_{t-1}^k = f_k z_t^k + const$ by Bayes' rule where $f_k = \frac{\sigma_k^{-2}}{\sigma_k^{-2}+\sigma_{k0}^{-2}}$ is the weight given to the private signal received by the firm and $1-f_k$ is the weight given to the public information $\rho_k z_{t-1}^k$ inferred from the law of motion of the underlying shocks k. If there was no dispersed information, and the firms knew the shock k perfectly, i.e, the case of perfect information, then $f_k = 1$.

Ignoring constants

$$\frac{a_\pi^d z_t^d + a_\pi^s z_t^s}{(1-\theta)} = f_d z_t^d \times [(1-\beta\theta)(\gamma+\psi)a_y^d + a_\pi^d + \beta\theta/(1-\theta)\rho_d a_\pi^d] \tag{25}$$

$$+ f_s z_t^s \times [(1-\beta\theta)\{(\gamma+\psi)a_y^s + 1\} + a_\pi^s + \beta\theta/(1-\theta)\rho_s a_\pi^s] \tag{26}$$

Comparing coefficients of the shocks

$$\frac{a_\pi^d}{(1-\theta)} = f_d * [(1-\beta\theta)(\gamma+\psi)a_y^d + a_\pi^d + \beta\theta/(1-\theta)\rho_d a_\pi^d]$$

$$\frac{a_\pi^s}{(1-\theta)} = f_s * [(1-\beta\theta)\{(\gamma+\psi)a_y^s + 1\} + a_\pi^s + \beta\theta/(1-\theta)\rho_s a_\pi^s]$$

Let

$$A_k = \frac{1}{1-\theta} - f_k(1 + \frac{\beta\theta\rho_k}{(1-\theta)}) \tag{27}$$

$$B_k = f_k \times [(1-\beta\theta)(\gamma+\psi) \tag{28}$$

Then two crucial equations

$$A_d a_\pi^d = B_d a_y^d \tag{29}$$

$$A_s a_\pi^s = B_s a_y^s + B_s/(\gamma+\psi) \tag{30}$$

From Euler and Taylor we get

$$y_t = E_t y_{t+1} - \frac{\phi_\pi^{Tay}\pi_t + \phi_y^{Tay}(y_t + z_t^s/(\gamma+\psi)) - E_t\pi_{t+1}}{\gamma} + z_t^d$$

$$C_k = 1 - \rho_k + \frac{\phi_y^{Tay}}{\gamma} \tag{31}$$

$$D_k = -\frac{\phi_\pi^{Tay} - \rho_k}{\gamma} \tag{32}$$

Then next two crucial equations

$$C_d a_d^y = D_d a_d^\pi + 1 \tag{33}$$

$$C_s a_s^y = D_s a_s^\pi - \frac{\phi_y^{Tay}}{(\gamma+\psi)\gamma} \tag{34}$$

From 29,

$$a_\pi^d = A_d^{-1} B_d a_y^d$$

substituting in 33 we get

Final 1,2

$$a_d^y = [C_d - D_d A_d^{-1} B_d]^{-1}$$

$$a_d^\pi = A_d^{-1} B_d [C_d - D_d A_d^{-1} B_d]^{-1}$$

From 34 and 30

$$C_s a_s^y = D_s A_s^{-1}(B_s a_y^s + B_s/(\gamma+\psi)) - \frac{\phi_y^{Tay}}{(\gamma+\psi)\gamma}$$

Final 3,4

$$a_s^y = [C_s - D_s A_s^{-1} B_s]^{-1}\{\frac{D_s A_s^{-1} B_s}{\gamma+\psi} - \frac{\phi_y^{Tay}}{(\gamma+\psi)\gamma}\}$$

$$a_\pi^s = A_s^{-1} B_s a_y^s + \frac{A_s^{-1} B_s}{\gamma+\psi}$$

Now, let $K_k = [C_k - D_k A_k^{-1} B_k]^{-1}$. After some algebra,

$$a_i^d = (\phi_\pi^{Tay} A_d^{-1} B_d + \phi_y^{Tay}) K_d \tag{35}$$

$$a_r^d = ((\phi_\pi^{Tay} - \rho_d) A_d^{-1} B_d + \phi_y^{Tay}) K_d \tag{36}$$

$$a_i^s = (\phi_\pi^{Tay} A_s^{-1} B_s + \phi_y^{Tay}) K_s \frac{1-\rho_s}{\gamma+\psi} \tag{37}$$

$$a_r^s = ((\phi_\pi^{Tay} - \rho_s) A_s^{-1} B_s + \phi_y^{Tay}) K_s \frac{1-\rho_s}{\gamma+\psi} \tag{38}$$

Thus Lemma 1 is proved.

A.2 Share of demand in unexpected inflation

Let $V_k = \sigma_h^{2k}/\sigma_h^{2\pi}$ for $k \in (d,s)$. In the data we look at 2 year ahead nominal yields or interest rates, and since the model is at monthly frequency, 14,we get

$$\frac{\Delta i_{2yr}^{obs}}{surprise} = \rho_d^{24} a_i^d a_\pi^d V_d + \rho_s^{24} a_i^s a_\pi^s V_s$$

Similarly, by 15, 2 year ahead consumption expectations are given by

$$\frac{\Delta c_{2yr}^{obs}}{surprise} = \rho_d^{24} a_c^d a_\pi^d V_d + \rho_s^{24} a_c^s a_\pi^s V_s$$

Solving linear system of 2 equations and 2 variables we get:

$$\rho_d^{24} a_\pi^d V_d = \frac{\frac{\Delta i_{2yr}^{obs}}{surprise} a_c^s - \frac{\Delta c_{2yr}^{obs}}{surprise} a_i^s}{a_i^d a_c^s - a_c^d a_i^s}$$

and

$$\rho_s^{24} a_\pi^s V_s = \frac{-\frac{\Delta i_{2yr}^{obs}}{surprise} a_c^d + \frac{\Delta c_{2yr}^{obs}}{surprise} a_i^d}{a_i^d a_c^s - a_c^d a_i^s}$$

$$\begin{aligned}
\text{Share}_{dd} &= \frac{a_\pi^{2d}\sigma_h^{2d}}{a_\pi^{2d}\sigma_h^{2d} + a_\pi^{2s}\sigma_h^{2s}} \\
&= \frac{a_\pi^{2d} V_d}{a_\pi^{2d} V_d + a_\pi^{2s} V_s} \\
&= \frac{a_\pi^d/\rho_d^{24}\,\frac{\frac{\Delta i_{2yr}^{obs}}{surprise} a_c^s - \frac{\Delta c_{2yr}^{obs}}{surprise} a_i^s}{a_i^d a_c^s - a_c^d a_i^s}}{a_\pi^d/\rho_d^{24}\,\frac{\frac{\Delta i_{2yr}^{obs}}{surprise} a_c^s - \frac{\Delta c_{2yr}^{obs}}{surprise} a_i^s}{a_i^d a_c^s - a_c^d a_i^s} + a_\pi^s/\rho_s^{24}\,\frac{\frac{-\Delta i_{2yr}^{obs}}{surprise} a_c^d + \frac{\Delta c_{2yr}^{obs}}{surprise} a_i^d}{a_i^d a_c^s - a_c^d a_i^s}} \\
&= \frac{\frac{a_\pi^d}{\rho_d^{24}}\left(a_c^s - a_i^s \frac{\Delta c_{2yr}^{obs}}{\Delta i_{2yr}^{obs}}\right)}{\frac{a_\pi^d}{\rho_d^{24}}\left(a_c^s - a_i^s \frac{\Delta c_{2yr}^{obs}}{\Delta i_{2yr}^{obs}}\right) + \frac{a_\pi^s}{\rho_s^{24}}\left(-a_c^d + a_i^d \frac{\Delta c_{2yr}^{obs}}{\Delta i_{2yr}^{obs}}\right)}
\end{aligned}$$

Thus, share of demand in unexpected inflation depends only on calibrated a ρ and $\Delta c_{2yr}^{obs}/\Delta i_{2yr}^{obs}$.

$$\text{Share}_{dd} = \frac{\frac{a_\pi^d}{\rho_d^{24}}\left(a_c^s - a_i^s \frac{\Delta c_{2yr}^{obs}}{\Delta i_{2yr}^{obs}}\right)}{\frac{a_\pi^d}{\rho_d^{24}}\left(a_c^s - a_i^s \frac{\Delta c_{2yr}^{obs}}{\Delta i_{2yr}^{obs}}\right) + \frac{a_\pi^s}{\rho_s^{24}}\left(-a_c^d + a_i^d \frac{\Delta c_{2yr}^{obs}}{\Delta i_{2yr}^{obs}}\right)}$$

Share of dd increases with $\Delta c_{2yr}^{obs}/\Delta i_{2yr}^{obs}$. The derivative of Share_{dd} with respect to $\Delta c_{2yr}^{obs}/\Delta i_{2yr}^{obs}$ is greater than zero given we know that $a_i^d, a_i^s, a_\pi^d, a_\pi^s, a_c^d, \rho_d, \rho_s > 0$ and $a_c^s < 0$.